VIGNETTES FROM THE WORLD OF GRAVITATIONAL WAVE ASTROPHYSICS

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Storyline

- Remember gravitational waves
- The Gravitational Wave Enterprise

1.5 vignettes

- Eccentric Binaries & Harmonic Correlation (w/ Ron Hellings)
- Briefly: Neutron Stars and the Equation of State

- Challenges for Gravitational Wave Astrophysics
The Cosmos as we know it

- **Light** has been our messenger from the Universe
A myriad of instruments exist to detect photons, but photons are LAME. Photons are easily distracted by pretty bits of matter they encounter on their way to Earth.
Since photons are **lame**, look at the Cosmos in a different way

Don’t look with **light**, look with **gravity**

- Gravitational wave astrophysics detects ripples in the fabric of spacetime that carry detailed information about the **dynamic motion** of matter and energy in the Cosmos
Foundations of Gravitational Wave Astronomy

- Gravitational wave astronomy, like most of modern astronomy, is highly **interdisciplinary**. There are three main thrusts:

  - **Technology**
  
  ![Technology Image]

  - **Science Analysis**
  
  ![Science Analysis Image]

  - **Astrophysics & Gravitational Science**
  
  ![Astrophysics Image]
Eccentric binaries

- Orbital eccentricity is something traditionally ignored because of a single paper: Peters & Mathews (1963)

- Gravitational wave emissions tend to circularize orbits

- Astrophysics shows that eccentricity can often persist, or be injected into systems (e.g. supernovae explosions)

- Can we detect it?

- Peters & Mathews to the rescue: gravitational wave power is encoded at harmonics of the orbital frequency

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**Gravitational Radiation from Point Masses in a Keplerian Orbit**

P. C. Peters* and J. Mathews

California Institute of Technology, Pasadena, California

(Received 18 January 1963)

The gravitational radiation from two point masses going around each other under their mutual gravitational influence is calculated. Two different methods are outlined; one involves a multiple expansion of the radiation field, while the other uses the inertial tensor of the source. The calculations apply for arbitrary eccentricity of the relative orbit, but assume orbital velocities are small. The total rate, angular distribution, and polarization of the radiated energy are discussed.

I. INTRODUCTION

The linearized version of Einstein's general theory of relativity is strikingly similar to classical electromagnetism. In particular, one might expect masses in arbitrary motion to radiate gravitational energy. The question has been raised, however, whether the energy so calculated has any physical meaning. We shall not concern ourselves with this question here; we shall take the point of view that the analogy with electromagnetic theory is a correct one, and energy is actually radiated.

In Sec. II we outline briefly two methods which can be used to calculate rates of emission of gravitational energy from a system of masses on which no net external force acts. Only enough details are presented to enable them to be applied to other problems; derivations and proofs are omitted. In Sec. III these methods are applied to obtain the total rate of radiation by two point masses going around each other in the familiar Kepler ellipse. In Sec. IV we discuss the angular distribution and polarization of the radiation.

II. GENERAL METHODS

A. Inertia Tensor

If one linearizes the equations of general relativity, setting $\varepsilon = \delta x + \alpha x$, $(|\alpha x| \ll 1)$, with $\alpha = 32\pi G$, one obtains

$$\varepsilon_{\mu\nu} = -\frac{1}{2} T_{\mu\nu},$$

where

$$h_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h_{\lambda\mu} h_{\lambda\nu},$$

and $T_{\mu\nu}$ is the total stress-momentum-energy tensor of the source, including the gravitational field stresses.

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* National Science Foundation Pre-Doctoral Fellow.

1 See, for example, L. Infeld and J. Pilchanski, Motion and Relativity (Pergamon Press Inc., New York, 1960).


3 R. P. Feynman, lectures, California Institute of Technology (unpublished).

4 Greek letters run from 1 to 4; $a, b, c, d, a, b, c, d$. Roman letters run from 1 to 3; $a, b, c, a, b$. The Kronecker delta $\delta_{\mu\nu}$ is $+1$ for $\mu = \nu = 1, 2, 3$, $-1$ for $\mu = 1, 2, 3, \nu = 1, 2, 3$. The phase of a plane wave is $h_{\mu\nu} = \omega t - k \cdot x$. $G$ is the usual gravitational constant in units of $6.67 \times 10^{-11}$ g cm$^{-3}$ s$^{-2}$.
Simulating binaries

- Don’t idealize binaries; simulate the binaries using the parameters astronomers are used to
  - **Orientation**: inclination, orbital phase, ascending node
  - **Size**: semi-major axis, orbital period, masses
  - **Sky Location**

- Wahlquist (1987) has worked out the quadrupolar waveforms in the context of these parameters in terms of $h_+$ and $h_\times$
Simulated binary

- Consider a binary with moderate eccentricity
- Compute the gravitational waveforms

\[
\begin{align*}
a \text{ (AU)} &= 0.00112023 \\
ecc &= 0.5 \\
\text{inc (deg)} &= 30.0 \\
m1 \text{ (msun)} &= 0.7 \\
m2 \text{ (msun)} &= 0.7 \\
R \text{ (pc)} &= 100.0 \\
\text{tsim (yr)} &= 0.0155756615963
\end{align*}
\]

Orb Period = 1000 sec
Time samples = 15 sec
Simulated Waveforms

- Add Gaussian noise to the waveforms at 10x the maximum amplitude of the signal we are looking for.
Fourier Transform of the data

- The zeroth order search for binaries in data is to take a Fourier Transform and look for periodicity. *Do you see the signal?*
The Harmonic Correlation Concept

- Take a time series waveform from Wahlquist, with added Gaussian noise
- FFT, and construct the power at each frequency
- At each frequency, multiply the power by the power at each higher harmonic
- This is the “harmonic correlation”: \( H = P(f) \times P(2f) \times P(3f) \times ... \)
  - Effect is frequencies which have significant power at harmonics are amplified, whereas uncorrelated patterns with low power damp out

**Pitfalls:**

- Can’t multiply harmonics ad infinitum. The power gets small and kills advantage you gain ➔ Pick first few harmonics, with most of the power
- At high eccentricity, binary power is spread over more harmonics; their power is progressively weaker and more easily buried in noise
Apply the harmonic correlation, and the signal stands out!
The harmonics that give us the detectable signal in the harmonic correlation are **not prominent** in the FFT!
What should I make of this?

Shane & Ron are geniuses!

- Harmonic correlation is an effective technique for detecting eccentric binaries
- Simple, straightforward search technique requiring only an FFT of the data
- The technique doesn’t characterize sources; it is good for discovery

Future Work

- Inclusion of instrumental noise (colored noise, eg. LISA)
- Extension to more extreme eccentricities (auto-correlation)
- Extension to EMRIs (e.g. using MLDC EMRI mock data)
Neutron Stars (briefly)

**WARNING:** Lots of handwaving and nebulous statements

- There are two end states that can emerge from the catastrophic end of stellar evolution (supernovae):
  - Neutron stars
  - Black holes
- Which stellar remnant is created depends on the mass of the progenitor
- *It is unknown what, if any, overlap there is between the resulting masses of neutron stars and black holes*
Stellar graveyard mass gap

- Neutron star masses seem to cluster around the Chandrasekhar mass
- Black hole masses seem to cluster at higher masses
- Do the mass ranges overlap?
- Is there a mass gap in the stellar graveyard, where no remnants exist?
- We can find out with gravitational waves

Figure from Clark et al. (A&A 392, 909 [2002])
Recognizing neutron stars

- Gravitational wave amplitudes **scale with mass**

\[ h_o = \frac{M_c}{D} \left( \pi f M_c \right)^{2/3} \]

- Any given signal could by any kind of massive object

- How do I tell neutron stars apart from black holes?

- Look for **tidal disruption** of the neutron star.

- When the star is destroyed, its gravitational wave signal vanishes

- A **direct measure** of the binding energy of the neutron star
There are many ways to treat fluids, depending on the physical situation of interest. Neutron stars are relativistic objects!
To study the mass gap

- **Blueprint:**
  - Treat the neutron star fluid relativistically
  - Determine the maximum mass of the neutron star
  - Simulate tidal disruptions using the relativistic fluids and maximum masses
  - Extract gravitational wave signals from the disruption simulations
  - Overlap the new signals with the old (currently used) templates to determine if we can tell the difference
Challenges for Gravitational Wave Astronomy

- Community Building & Communication
  - Because of the disparate groups involved in the enterprise, broadly trained people with the ability to communicate across discipline boundaries are highly valued.
  - Primary communication lines are between traditional gravitational theory, experimental physicists, technology professionals, and astrophysicists.

- Brain Power
  - The gravitational wave effort is extremely understaffed.
  - Need to have more professors in the field.
  - Need to train more young people in the field.
  - Need to retain young people in the field.