Ripples on a cosmic sea

Cutting-edge astronomy is making waves

by Shane L. Larson

HIGH ON THE COLUMBIA plateau of eastern Washington state, a remarkable astrophysical observatory is being constructed; its twin is taking shape in the lush forests of central Louisiana. If you're peering through the dust and the tumbleweeds (or the thick mossy forest), don't expect to see any domes housing massive optical assemblies or great radio dishes tracking across the skies. These observatories are not looking for the visible light from the countless burning stars throughout the Universe, nor for the faint radio whispers of charged particles thrashing about in their hot and violent environments. These are a phenomenal new kind of observatory called LIGO: the Laser Interferometer Gravitational-wave Observatory.

Gravitational waves were one of the novel predictions of Einstein's 1916 general theory of relativity, a completely new phenomenon that was not present in the Newtonian theory of gravity prevalent up to that time. It is only now, almost a century later, that technology has become sophisticated enough to possibly detect this new radiation, and observatories like LIGO are slowly taking shape across the planet.

The modern gravitational wave observatory is a large laser interferometer (typically with arms about 0.5 to 4 km long, for current designs). They are very similar to the familiar Michelson interferometer, but on a much larger and grander scale. Scientists will carefully monitor the output of the interferometers, looking for miniscule changes in the lengths of the interferometer arms, indicating the passage of a gravitational wave.

In addition to ground-based observations, scientists at NASA and the European Space Agency are also beginning to think about the search for gravitational waves in space. They are designing a much larger interferometer, known as LISA (Laser Interferometer Space Antenna), to be launched sometime late in the next decade. The mission will consist of three spacecraft arranged in a triangular constellation, 5 million kilometers per side. These three spacecraft will orbit the Sun in a triangular configuration, just over 52 million kilometers behind the Earth in its orbit, and inclined to the Earth's orbit by 60 degrees (figure 1). By monitoring laser signals exchanged between each of the spacecraft, scientists can monitor any change in distance between the craft in an effort to detect gravitational waves.

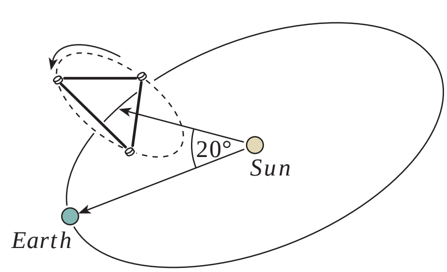

Figure 1

But what *are* gravitational waves? Why didn't we know about them before Einstein and why are they hard to detect? To understand this, we must explore the differences between Isaac Newton's theory of gravity and Albert Einstein's theory of general relativity.

Gravity according to Isaac

When Isaac Newton sat beneath the proverbial apple tree waiting for his fruitful concussion, his perception of the cosmos was built around the idea that space and time were immutable qualities of the Universe. From his perception (and indeed, from the perception of essentially all experimental evidence available at the time), space and time were fixed, absolute entities throughout the Universe.

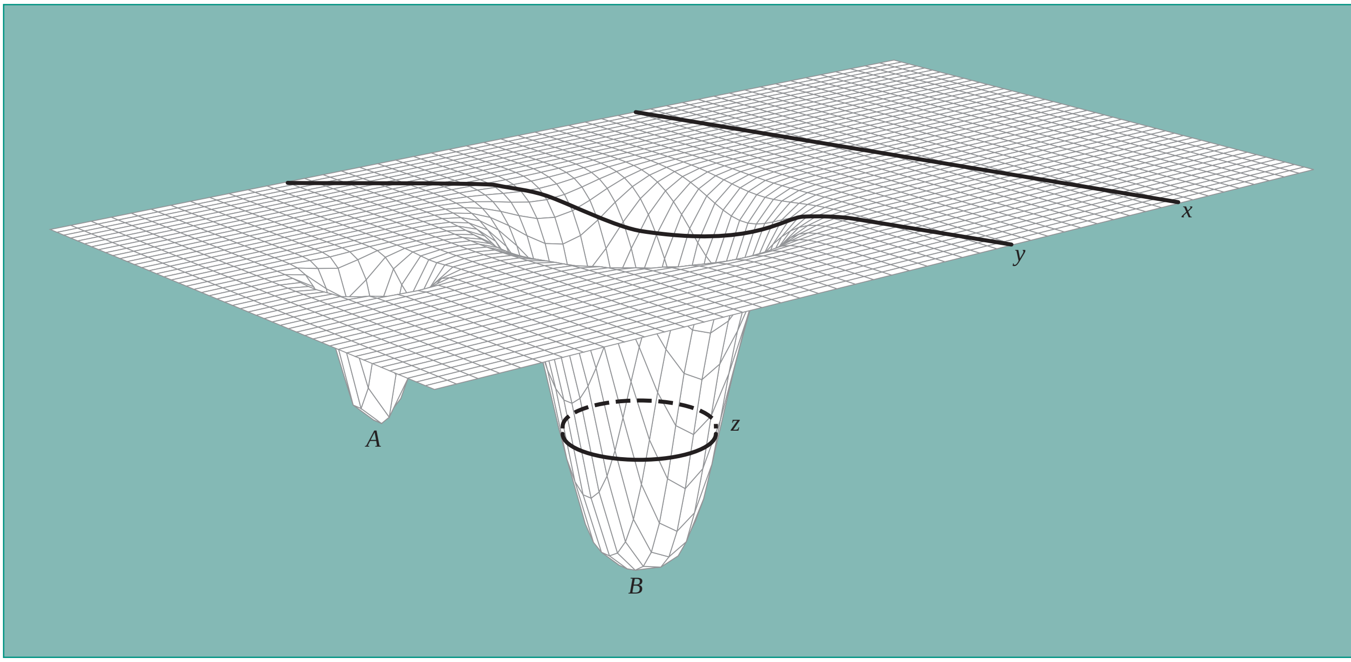

Figure 2

Newton set down his ideas about absolute space, time, and motion in 1687 in his monumental work, *Philosophiae Naturalis Principia Mathematica*, or "The Mathematical Principles of Natural Philosophy" (the *Principia* also covers Newton's ideas of *relative* space and time, which are defined with respect to human constructs; for example, time is measured in hours and minutes, and space is broken into distances or orientations with respect to Earth).

The idea that space and time were immutable, *universal* quantities allowed Newton to propose his very successful universal law of gravitation, which is familiar to us all:

$$F = \frac{G m_1 m_2}{r^2}. \tag{1}$$

The universality of space and time allowed the application of equation (1) to any gravitational system that could be observed, including the Moon, the planets, and even the newly discovered Galilean satellites of Jupiter (Galileo had detected the four largest satellites of Jupiter with his telescope in 1610), despite the fact that these systems were far removed from the time and space that could be sampled directly in any Earth-bound laboratory.

The theory was deceptively simple in its formulation and unprecedented in its predictive power. In particular, it can be used to construct (and indeed was developed to explain) Kepler's laws of planetary motion; this can be easily demonstrated for the special case of circular orbits if one assumes equation (1) to be the centripetal force that binds a mass (for example, the mass of an asteroid, or other small test mass) in its orbit about the Sun.

An important aspect of Newtonian gravity is that the gravitational force acts instantaneously (that is, changes in the gravitational field propagate at infinite speed). If the mass of the Sun were to suddenly change, or if it were to start moving away from its location at the center of the Solar System, each of the planets would know instantly and their orbits would change in accordance with the new configuration of the gravitational field.

Gravity according to Albert

The Newtonian theory of gravity was a cornerstone of physics for more than 200 years, and even today it is extraordinarily useful and particularly well suited for many different applications, such as computing spacecraft trajectories or describing the orbits of binary stars. But when Einstein published his special theory of relativity in 1905, it immediately began to pose problems for the Newtonian theory of gravity.

Special relativity introduced many wondrous and strange predictions about the relationships between moving clocks and rulers, but one of its most important predictions is the existence of a universal speed limit: $c = 3.0 \times 10^8$ m/s. Nothing can travel faster than the speed of light. In contrast, Newton's theory of gravity allows infinite propagation speeds, clearly a violation of the much slower speed limit c.

Einstein set out to formulate a theory of gravity consistent with special relativity, and in 1916 published his *general theory of relativity*. General relativity breaks with Newtonian gravity from the outset by discarding the idea of the gravitational field in favor of a new concept: space-time geometry. Einstein's basic premise was that the motions of particles were not affected by an unseen force tugging on them, pulling them toward massive bodies. The motions of particles are determined by the geometry of the space-time around them.

A visual analogue of Einstein's remarkable idea may be seen in figure 2, which illustrates the "rubber-sheet" model of general relativity. Space-time is "flat" when there is no mass present, as shown at the extreme right edge of the surface. The presence of a massive body curves space-time, as shown at point A (imagine placing a small lead weight on the rubber sheet). More massive bodies produce more curvature in space-time, deforming it more than smaller masses (imagine placing a bowling ball at point B).

How does the shape of space-time affect the motion of particles? Consider the three trajectories labeled x, y, and z shown in figure 2. You can imagine that each of these paths is the trajectory of a Ping-Pong ball rolling across the rubber sheet; they are analogous to the paths of particles (such as satellites, asteroids, and comets) in the vicinity of massive bodies. The path x represents the path of a particle through space-time when it is far from any mass. Such a path is called a *geodesic*. In this rubber-sheet model, geodesics are the shortest length paths between any two points. In the flat regions of the sheet, the geodesics are familiar straight-line paths.

There are other "straight-line" paths in space-time, such as the trajectory y. Imagine a ball rolling along y, which is initially parallel to x. When the ball encounters the curved region of space-time, the geodesic path dips into the curved region, and reemerges along a new direction that is diverging from x. This path is also a geodesic because it is the straightest trajectory for the ball through the region of high curvature. No external forces acted on the ball to alter its trajectory. The trajectory was altered only by virtue of the fact that the ball rolled on a curved surface.

Now think about the curve z. This path is also a geodesic; the ball rolls along its trajectory, free of external forces pushing or pulling on it, its course determined only by the curvature of the space around it.

Each of the three paths in figure 2 is analogous to familiar particle trajectories described in terms of a central potential,

$$V = -\frac{GM}{r}, \qquad (2)$$

where M is the mass of the central source of the potential. The path x is that followed by a particle far from any source of gravitational attraction, the path y is that of a particle scattering off a gravitational potential, and the path z is that of a particle in orbit about a larger mass.

This way of thinking about general relativity can be summarized in the two-line mantra of the modern gravitational theorist, popularized by Misner, Thorne and Wheeler in their classic text *Gravitation*: "Matter tells space how to curve; space tells matter how to move."

The idea that the "gravitational field" is simply curvature of space-time will be integral to our physical picture of a gravitational wave.

Gravitational waves

The existence of a cosmic speed limit is at great odds with Newtonian gravity, which allowed signals to propagate at infinite speed. By imposing the constraints of special relativity on a theory of gravity, we suddenly find a myriad of new phenomena we can experimentally search for in nature—phenomena that we did not know existed be-

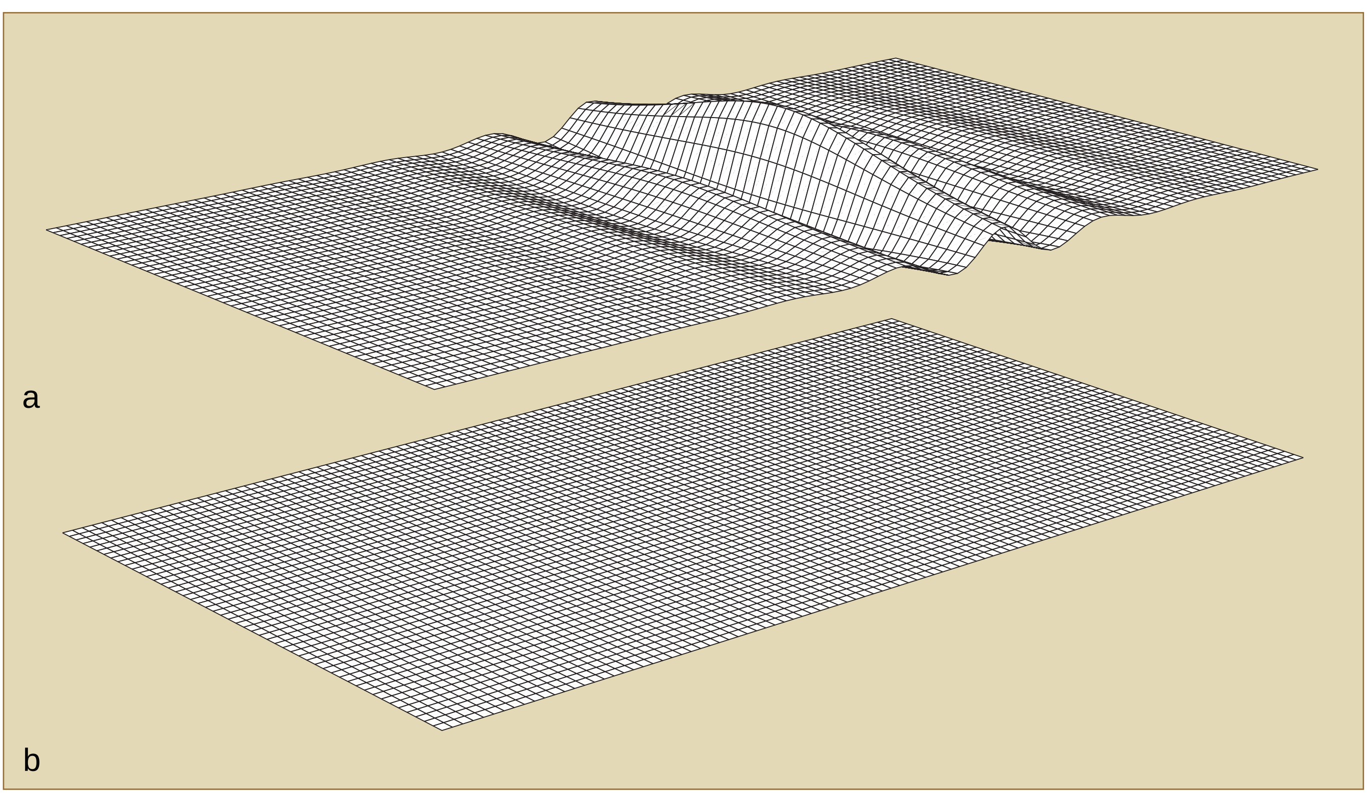

Figure 3

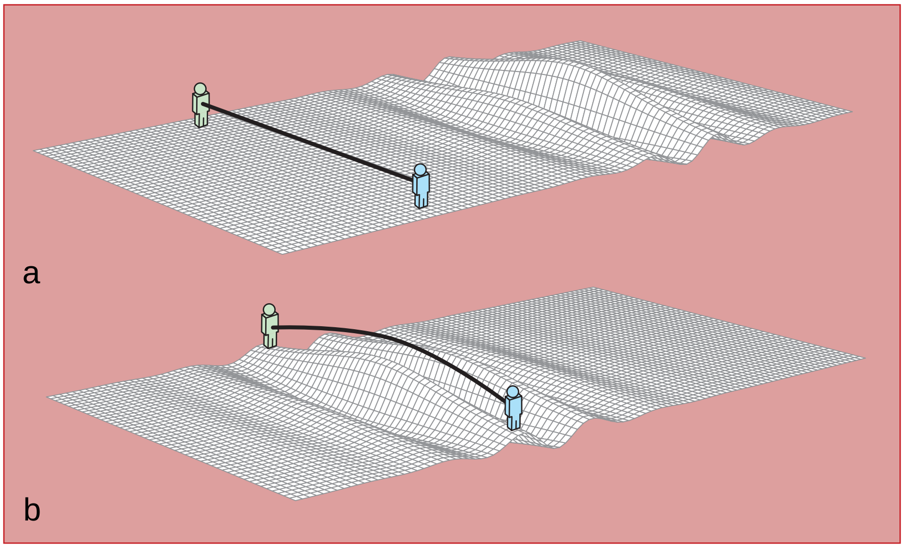

Figure 4

cause they simply cannot be explained with Newtonian physics. One example is the famous "bending of light" by a gravitational field, which Einstein put forth as a test of his new theory of gravity. Eddington's measurement of the deflection of starlight by the Sun during the total eclipse of 1919 confirmed the predictions of general relativity and made Einstein a worldwide celebrity.

To understand how waves are treated by modern relativistic theory, recall our Einsteinian description of gravity as curvature of space-time. If the analogue to the gravitational field is curvature, then changes in the gravitational field are analogous to changes in the curvature of space-time. When changes in curvature propagate, moving through space-time, they are called gravitational waves. Figure 3 shows a model of gravitational waves in the context of the rubber-sheet analogue outlined in the section above.

Like the waves we are more familiar with, gravitational waves have an amplitude (usually denoted h), a wavelength λ, and a frequency f, which are related to the propagation speed c:

$$c = \lambda f. \qquad (3)$$

It is no accident that the propagation speed is written as c; general relativity predicts that gravitational waves travel at the speed of light.

Detection

How does one go about detecting gravitational waves? To do this, you must develop a way to measure the changes in the space-time curvature. We can imagine a detector of gravitational waves in the rubber-sheet model we developed above. Consider the two people in figure 4a. They are hanging out in an essentially flat space-time, shining a flashlight back and forth at each other, and timing how long it takes the beam to traverse the distance between them. This time is a measure of the *proper distance* between them. Unbeknownst to them, a curvature wave is approaching, and it will affect the results of their experiment.

In figure 4b, the wave is upon our space-time experimenters. Because the wave has changed the curvature of the space-time between them, it takes a different amount of time for the photons to travel back and forth; our intrepid young experimenters can measure this time difference, thus detecting the wave!

Interferometers detect gravitational waves in much the same way, by comparing the distance along two different directions in spacetime. Using laser light, the beams in two different directions are interfered with each other. When a gravitational wave passes by, the lengths of the interferometer arms change, and so the interference pattern made by the two laser beams shifts.

The quantity measured in a gravitational wave observatory is called the *strain* and is defined as

$$s = \frac{\Delta l}{l}, \qquad (4)$$

where Δl is the change in proper length the gravitational wave produces between our two experimenters and l is the unperturbed length, before the wave is upon them. The strain can be approximately related to the amplitude of the wave by $s \sim h/2$.

Laser interferometers aren't the only way to detect gravitational waves. One could imagine that the two experimenters in figure 4 aren't shining a flashlight back and forth, but rather are holding a long metal bar between them. When the gravitational wave passes by, it stretches the bar a little. The bar snaps back to its original shape after the wave passes by, and as a consequence begins to "ring" (that is, it begins to vibrate). The frequencies that the bar can see depend on its length. Roughly speaking, the bar is sensitive to waves that have a frequency corresponding to its normal modes of vibration:

$$f = n\frac{v}{2l}, \qquad (5)$$

where v is the speed of sound in the bar, l is the length of the bar, and n is an integer indicating the mode. The amplitude of these oscillations depends on the strain induced in the bar by the gravitational wave.

These types of detectors are called "bar-detectors" and were first pioneered by Joseph Weber at the University of Maryland in the 1960s. The most sensitive bar-detector in operation today is called ALLEGRO, and is operated by Louisiana State University.

Sources

Gravitational waves are created from the dynamical motions of mass. Any asymmetric acceleration in a massive system will generate gravitational waves (to be precise, a

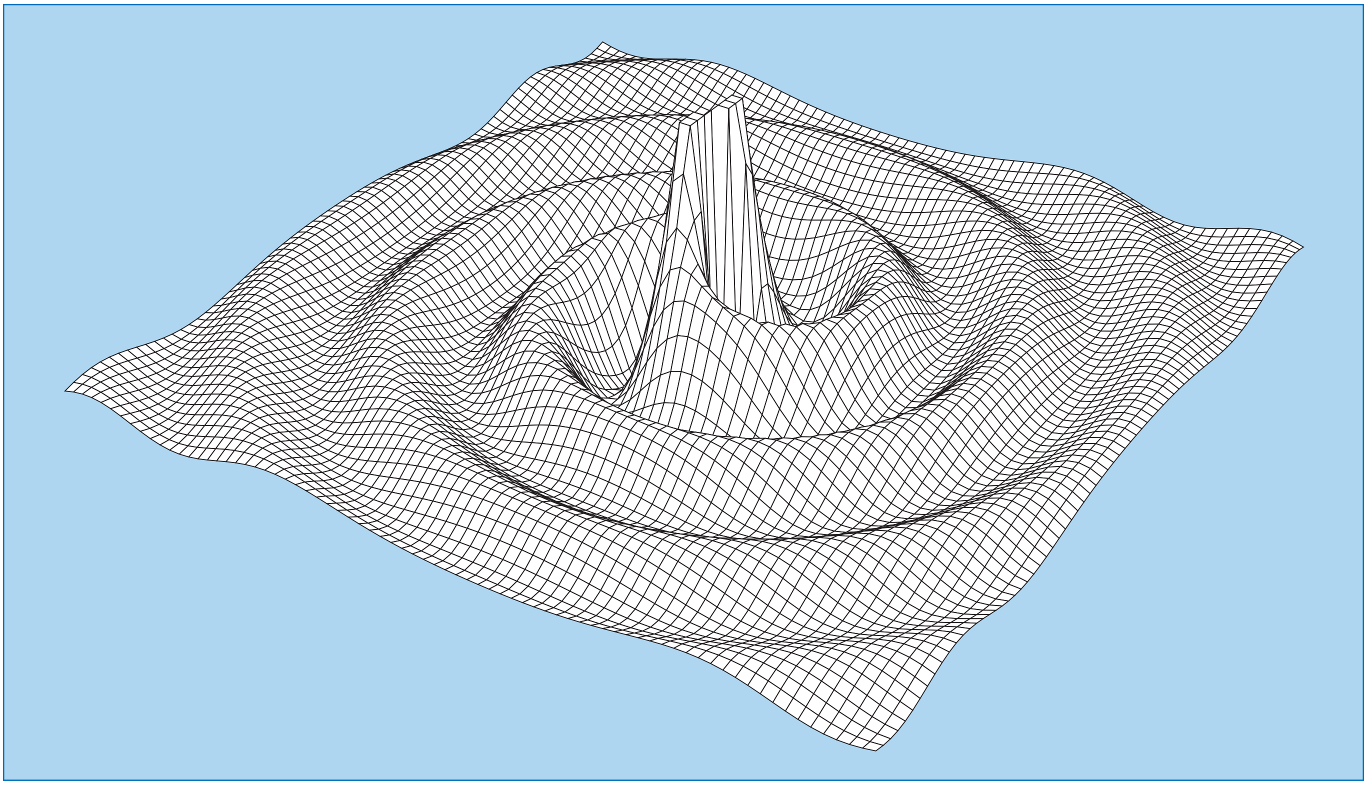

Figure 5

system will emit gravitational radiation if it has a non-zero *quadrupole moment*). Symmetric motions, such as radial pulsations in spherical stars, will not generate gravitational waves.

Purely symmetric systems are, of course, an ideal in physics and very unlikely to exist in nature. A survey of common astrophysical systems shows that the cosmos is replete with asymmetric dynamical systems, from active galactic nuclei and spiral galaxies on the largest scales to supernovae and common binary stars on smaller scales.

Binary systems in general are expected to be one of the most important sources of continuous astrophysical gravitational waves. Interesting targets for study include binary stars, neutron-star binaries, black hole binaries, or combinations of any of the three. Throughout most of their lives, binary systems evolve slowly. If they are in approximately circular orbits, gravitational waves are emitted with a frequency that is twice the orbital frequency:

$$f = 2f_{\mathrm{orb}}. \qquad (6)$$

These waves are said to be *monochromatic* in analogy with single frequency visible light. Figure 5 shows the amplitude of the gravitational waves generated by a typical binary system. The stars generating the radiation lie at the center of the figure. Gravitational waves are thrown off as a result of their orbital motion, and propagate out through the vast sea of space-time until they come gently lapping up on the shores of Earth.

The (dimensionless) amplitude of gravitational waves radiated by a binary system, as measured at the Earth, can be estimated by the formula

$$|h| = \frac{4G^2 m_1 m_2}{c^4 a R}, \qquad (7)$$

where m_1 and m_2 are the masses of the binary components, G is Newton's constant, a is the semi-major axis of the binary orbit, R is the distance from the Earth to the binary, and c is the speed of light. By rewriting a using Kepler's third law

$$a^3 = \tau^2 \frac{G(m_1 + m_2)}{4\pi^2}, \qquad (8)$$

equation (7) may be expressed in terms of the orbital frequency (which is related to the gravitational wave frequency by equation (6)):

$$|h| = \frac{2\pi^{2/3}(2G)^{5/3}}{c^4 R} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}} f_{orb}^{2/3}. \qquad (9)$$

Indirect, astrophysical evidence for the existence of gravitational waves exists from monitoring the famous Hulse-Taylor binary pulsar, PSR 1913+16, over the past 25 years. By monitoring the pulsar's orbit, it was discovered that the orbital period was gradually shrinking, implying the two pulsars are slowly spiraling together. The rate at which the orbital period is changing is *precisely* the amount that general relativity predicts through the emission of gravitational radiation. Gravitational waves from the orbital motion of the pulsars carry away orbital energy, causing the orbit to shrink. This discovery earned Joseph Taylor and Russell Hulse the 1993 Nobel Prize in physics.

We can estimate the amplitude of the gravitational waves from this pulsar when they arrive at the Earth using equation (9). This binary has two stars, each of mass $m \sim 1.4 M_{sun}$, an orbital period of about 7.75 hours (implying $f_{orb} \sim 3.58 \times 10^{-5}$ Hz), and lies at a distance of $r \sim 5$ kpc from Earth. This gives a dimensionless amplitude of $|h| = 6.4 \times 10^{-23}$, a very small amplitude indeed! This is many orders of magnitude lower than the minimum amplitude detectable by LISA at this frequency.

Is it possible for us to detect sources of gravitational radiation much closer to the Earth? For instance, binaries are expected to be good sources of gravitational waves; might we expect to see gravitational radiation from the orbits of the Galilean satellites (Io, Ganymede, Callisto, and Europa) about Jupiter? Consider equation (9) again, and let's apply it to the case of Io, the innermost of the Galilean satellites. Jupiter has a mass of 1.90×10^{27} kg, and Io has a mass of 8.94×10^{22} kg. The orbital period of Io is 1.77 days, giving an orbital frequency of $f_{orb} = 6.54 \times 10^{-6}$ Hz. At their closest approach, Jupiter and the Earth are 6.29×10^{8} km apart. Using equation (9) with these values gives an amplitude of $|h| = 1.4 \times 10^{-24}$. Despite being significantly closer to Earth than the binary pulsar, the gravitational radiation from Io is much weaker. It is not expected that detectors such as LIGO or LISA will detect any gravitational radiation from any source within our own Solar System.

What we hope to learn

Gravitational waves are a completely new way of looking at the Universe. When the first radio telescopes were built, we learned a tremendous amount about distant astrophysical systems because radio waves bring us different information than ordinary light. Similarly, we hope that by observing the Universe in gravitational waves, we should learn different things than we would by looking in ordinary light. In particular, we should be able to observe the collisions of massive black holes, the collisions of neutron stars, stars falling into the black holes at the centers of galaxies, and supernova explosions.

The most prevalent type of source will be close binaries such as those described above. Early on, close binary systems will have relatively small orbital frequencies. Frequencies in the range of roughly 10^{-5} to 10^{-1} Hz should be accessible to LISA. Late in their lives, binary systems tend to evolve rapidly, the components spiraling toward one another. As they spiral together, the frequency ramps up rapidly and the binary "chirps." Ultimately, these binary systems coalesce to form a single object. This high frequency inspiral, chirp, and coalescence should be observable by LIGO at frequencies from about 10 to 1000 Hz.

The process of coalescence will be a dynamic and violent one, and scientists expect it to produce copious amounts of gravitational radiation. By studying these gravitational waves, it is hoped we will gain our first direct observations of what happens during the collision of two massive bodies and how the final object wobbles, stretches, and vibrates before settling down into its final state. Short, transient pulses of gravitational radiation, such as supernovae, are known as *burst sources*. Predicting what bursts of radiation from violent events might look like using a gravitational wave observatory such as LIGO or LISA is a problem that is at the forefront of modern theoretical physics, and is being studied using advanced numerical simulations on fast-computing systems. Whether or not we will be able to detect burst sources (they are much harder to detect than inspiraling binaries) will depend on precisely how strong the burst of radiation from an explosive event is, and how far away from Earth it is.

The future

Because of the weak nature of gravitational waves, it is possible that our initial searches with LIGO will not detect any gravitational radiation. This is largely due to limitations in technology, but as computing power, laser technology, and our understanding of gravitational waves improves, we'll be able to build better gravitational wave observatories. Plans to upgrade LIGO to LIGO II are already in place, and should make the detection of gravitational waves a routine occurrence.

Space-based observatories such as LISA are literally guaranteed to see nearby interacting white dwarf binary stars. The closest of these, a star called AM CVn, is a helium cataclysmic variable about 100 parsecs away in the constellation of Canes Venatici, and can be seen in small telescopes. Since stars like AM CVn can be observed with ordinary telescopes, we know a tremendous amount about the masses and orbits of these binary systems. Therefore, we know what the gravitational wave signal should look like, and should be able to detect such stars almost immediately after LISA becomes operational.

The study of gravitational waves from the Universe at large promises to produce a revolution in astrophysics as spectacular as the revolution brought on by the advent of X-ray, radio, and γ-ray astronomy. Unlike photons, gravitational waves propagate very readily through regions of dense gas and dust. Gravitational waves will be generated by the mysterious "dark matter" that seems to pervade much of the cosmos, and should have been generated in the earliest moments after the Big Bang. By studying this remarkable new type of radiation, astrophysicists will, for the first time, be able to probe the dense cores of galaxies, see the inspiral and collision of neutron stars millions of light years away, and study the region very near the event horizons of black holes. Gravitational wave astronomy promises to be one of the hottest areas of research as we move into the 21st century.

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