

# Gravitational wave bursts from the Galactic massive black hole

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## ABSTRACT

The Galactic massive black hole (MBH), with a mass of  $M_{\bullet} = 3.6 \times 10^6 M_{\odot}$ , is the closest known MBH, at a distance of only 8 kpc. The proximity of this MBH makes it possible to observe gravitational waves (GWs) from stars with periaapse in the observational frequency window of the *Laser Interferometer Space Antenna* (*LISA*). This is possible even if the orbit of the star is very eccentric, so that the orbital frequency is many orders of magnitude below the *LISA* frequency window, as suggested by Rubbo, Holley-Bockelmann & Finn (2006). Here we give an analytical estimate of the detection rate of such GW bursts. The burst rate is critically sensitive to the inner cut-off of the stellar density profile. Our model accounts for mass segregation and for the physics determining the inner radius of the cusp, such as stellar collisions, energy dissipation by GW emission and consequences of the finite number of stars. We find that stellar BHs have a burst rate of the order of  $1 \text{ yr}^{-1}$ , while the rate is of the order of  $\lesssim 0.1 \text{ yr}^{-1}$  for main-sequence stars and white dwarfs. These analytical estimates are supported by a series of Monte Carlo samplings of the expected distribution of stars around the Galactic MBH, which yield the full probability distribution for the rates. We estimate that no burst will be observable from the Virgo cluster.

**Key words:** black hole physics – gravitational waves – stellar dynamics – Galaxy: centre.

## 1 INTRODUCTION

When a star comes near the event horizon of a massive black hole (MBH) with mass  $M_{\bullet} \lesssim$  few times  $10^6 M_{\odot}$ , it emits gravitational waves (GWs) with frequencies observable by the planned *Laser Interferometer Space Antenna* (*LISA*). Such extreme mass ratio inspiral sources (EMRIs) can be observed by *LISA* to cosmological distances, provided that they spend their entire orbit emitting GWs in the *LISA* frequency band (Finn & Thorne 2000; Barack & Cutler 2004b,a; Gair et al. 2004; Glampedakis 2005). For an EMRI to be in the *LISA* band, the orbital period of the star has to be shorter than  $P \sim 10^4$  s. The formation mechanism for EMRIs begins when a star that is initially not strongly bound to the MBH is scattered to a highly eccentric orbit, such that its periaapse comes close to the Schwarzschild radius  $r_s$  of the MBH. The star loses energy to GWs on every orbit, slowly spiralling inward. This process may eventually lead to a closely bound orbit that is observable by *LISA*. Inspiral is often frustrated by scattering with other field stars (Alexander & Hopman 2003; Hopman & Alexander 2005), and the rate at which stars manage to spiral in successfully is rather low, of the order of  $0.1 \text{ Myr}^{-1}$  per Galaxy for stellar BHs (Hils & Bender 1995; Sigurdsson & Rees 1997; Miralda-Escudé & Gould 2000; Freitag

2001; Ivanov 2002; Freitag 2003; Hopman & Alexander 2006a,b; see Hopman 2006, for a review). However, due to the large distances ( $\sim 1$  Gpc) to which EMRIs can be observed, the integrated rate over the volume makes these a very promising target for *LISA*.

Our own Galactic Centre (GC) contains an MBH of  $M_{\bullet} = (3.6 \pm 0.3) \times 10^6 M_{\odot}$  (Schödel et al. 2002; Alexander 2005; Eisenhauer et al. 2005), in the range of MBH masses that will be probed by *LISA*. Since the Galactic MBH is very close,  $d \approx 8$  kpc (Eisenhauer et al. 2005), the stars near the MBH can be studied in great detail (Schödel et al. 2002; Genzel et al. 2003; Ghez et al. 2003; Schödel et al. 2003; Ghez et al. 2005). The Galactic MBH and its stellar cluster are therefore very useful as a prototype for extragalactic nuclei, in particular in the study of EMRIs. For a review of stellar processes near MBHs, see Alexander (2005).

It is unclear whether our own GC can be observed as a continuous source of GWs. From the very low event rates this appears to be highly unlikely, but due to its proximity, waves with lower frequencies can be observed in the GC, and it was suggested by Freitag (2003) that a number of low-mass main-sequence (MS) stars may be observed in our own GC.

Another possibility was considered by Rubbo et al. (2006, hereafter RHBF06), who showed that even a single fly-by of a star near the MBH would be observable by *LISA* if it is sufficiently close, and they estimated that the event rate is high enough ( $\sim 15 \text{ yr}^{-1}$ ) that several detectable fly-bys would be observable during the *LISA*

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mission. The prospect of observing the Galactic MBH as a GW burster is very exciting: it would imply detection of GWs from an object that has been extensively studied in many electromagnetic wavelengths. Furthermore, as we point out in this paper, GW bursts are caused by stars very close to the MBH, and thus probe a region near the MBH which is not accessible observationally in a direct way by other means.

RHBF06 used a single-mass stellar model to study the GW burst rate. The rate is dominated by nearby stars, raising the question what determines the inner cut-off of the stellar cusp; RHBF06 assumed that the cusp extends all the way to the MBH. Here we re-address the event rate of such GW bursts in the GC. We consider the treatment of a multimass system, and account for the inner cut-off of the cusp.

This paper is organized as follows. In Section 2 we derive the minimal periastron a star needs to have to give an observable burst of GWs in the GC. In Section 3 we give an analytical expression for the event rate of GW bursts. The rate is dominated by stars at very close distances from the MBH, and we discuss several processes which may determine the inner cut-off of the density profile. Our analytical model is complemented by Monte Carlo realizations of stellar cusps (Section 4), which allow an accurate treatment of rare events where a single star produces a large number of bursts. The Monte Carlo samplings also yield the probability distribution of the event rates, in addition to the average event rate. In Section 5 we present our resulting rates, and in Section 6 we discuss and summarize our results.

## 2 DETECTION OF GRAVITATIONAL WAVES FROM THE GALACTIC CENTRE

For  $f < \text{mHz}$ , a good approximation of the sensitivity curve of *LISA* (Larson 2001) is given by

$$S(f) = S_0 \left( \frac{f}{\text{Hz}} \right)^{-4} \left( \frac{L}{5 \times 10^{11} \text{ cm}} \right)^{-1/2} \quad (f < \text{mHz}), \quad (1)$$

where  $S_0 = 6.16 \times 10^{-51} \text{ Hz}^{-1}$ ,  $f$  is the frequency of a GW and  $L$  is the arm length of *LISA* (Larson 2001).

The signal-to-noise ratio (S/N),  $\rho$ , can be approximated by (Finn & Thorne 2000, equation 2.2)

$$\rho \approx \frac{h \mathcal{N}_f^{1/2}}{f^{1/2} S^{1/2}}, \quad (2)$$

here  $\mathcal{N}_f$  is the number of cycles spent at a certain frequency  $f$ ; for GW bursts  $\mathcal{N}_f = 1$ . Furthermore,  $h$  is the strain, which is here approximated by the quadrupole estimate of a circular orbit (Finn & Thorne 2000, equation 3.13)

$$h = 2.28 \times 10^{-16} \left( \frac{d}{8 \text{ kpc}} \right)^{-1} \times \left( \frac{m}{M_\odot} \right) \left( \frac{M_\bullet}{3.6 \times 10^6 M_\odot} \right)^{2/3} \left( \frac{f}{\text{Hz}} \right)^{2/3}. \quad (3)$$

From equations (1)–(3), the minimal frequency necessary to measure a burst is then

$$f_{\text{burst}} = 4.3 \times 10^{-5} \text{ Hz} \rho^{6/13} \left( \frac{d}{8 \text{ kpc}} \right)^{6/13} \times \left( \frac{m}{M_\odot} \right)^{-6/13} \left( \frac{M_\bullet}{3.6 \times 10^6 M_\odot} \right)^{-4/13}, \quad (4)$$

where  $f = \sqrt{GM/r_p^3}$  is now the orbital frequency at periastron of an eccentric orbit. The corresponding periastron is

$$\left( \frac{r_p^{\text{burst}}}{r_s} \right) = 60 \rho^{-4/13} \left( \frac{d}{8 \text{ kpc}} \right)^{-4/13} \times \left( \frac{m}{M_\odot} \right)^{4/13} \left( \frac{M_\bullet}{3.6 \times 10^6 M_\odot} \right)^{-6/13}, \quad (5)$$

or  $r_p^{\text{burst}} \approx 2.1 \times 10^{-5} \text{ pc}$ . The corresponding angular momentum is

$$\left( \frac{J_{\text{burst}}}{J_{\text{LSO}}} \right)^2 = 7.5 \rho^{-4/13} \left( \frac{d}{8 \text{ kpc}} \right)^{-4/13} \left( \frac{m}{M_\odot} \right)^{4/13} \times \left( \frac{M_\bullet}{3.6 \times 10^6 M_\odot} \right)^{-6/13} \left( 2 - \frac{r_p^{\text{burst}}}{a} \right), \quad (6)$$

where  $J_{\text{LSO}}^2 = (4GM_\bullet/c)^2$  defines the last stable orbit.

The event rate (equation 8) is approximately proportional to  $J_{\text{burst}}^2$ . In this model, we note that neglecting the noise from Galactic white dwarf (WD) binaries can be justified by the fact that the noise is much smaller than the instrumental noise at  $f_{\text{burst}} = 3 \times 10^{-5} \text{ Hz}$  (Bender & Hils 1997). At higher frequencies,  $2 \times 10^{-4} \text{ Hz} \lesssim f \lesssim 3 \times 10^{-3} \text{ Hz}$  the S/N will increase in spite of the presence of Galactic noise, because of the larger GW amplitude at those frequencies.

## 3 AN ANALYTICAL MODEL FOR THE GRAVITATIONAL WAVE BURST RATE IN THE GALACTIC CENTRE

### 3.1 The gravitational wave burst rate in an isotropic distribution

The distribution of stars near an MBH is an important problem in stellar dynamics, and has been studied since the early 1970s (Peebles 1972). The MBH dominates the dynamics within the radius of influence,  $r_h = GM_\bullet/\sigma^2$ , where  $\sigma$  is the stellar velocity dispersion far away from the MBH. For the GC,  $r_h \approx 2 \text{ pc}$  (Hopman & Alexander 2006a). It was shown by Bahcall & Wolf (1976) that within  $r_h$ , the density distribution of a single-mass population of stars is very well approximated by a power law,  $n(r) \propto r^{-\alpha}$ , with  $\alpha = 7/4$ . These results, which were obtained by solving the Fokker–Planck equation in energy space, were later confirmed by  $N$ -body simulations (Baumgardt, Makino & Ebisuzaki 2004a; Preto, Merritt & Spurzem 2004) and Monte Carlo simulations (Freitag & Benz 2002).

Bahcall & Wolf (1977) studied the distribution of stars with different masses near an MBH, and showed that mass segregation leads to steeper distributions of the more massive species which sink to the centre due to dynamical friction. These results were confirmed and extended by Freitag, Amaro-Seoane & Kalogera (2006) and Hopman & Alexander (2006b) for a much wider range of masses. For simplicity we approximate the distributions as power laws, with different exponents for different species (symbolized by ‘ $M$ ’), such that  $n_M(r) \propto r^{-\alpha_M}$ . The values of  $\alpha_M$  will be discussed in Section 3.2.

We assume an isotropic density profile; the role of modifications of the distribution function (DF) by the loss-cone will be discussed in Section 3.3.5. For such a distribution, the number of stars  $n(a, dJ^2) da dJ^2$  in an element  $(a, a + da)$ ,  $(J^2, J^2 + dJ^2)$  is given by

$$n(a, J^2) da dJ^2 = (3 - \alpha_M) \frac{C_M N_h}{r_h} \left( \frac{a}{r_h} \right)^{2-\alpha_M} \frac{1}{J_c^2(a)} da dJ^2, \quad (7)$$

where  $J_c(a) = \sqrt{GM_\bullet a}$  is the circular angular momentum,  $N_h$  is the number of MS stars within  $r_h$  and  $C_M N_h$  the total number of stars of type  $M$  within  $r_h$  (so that for MS stars  $C_{MS} \equiv 1$ ). The rate per unit of logarithmic of the semimajor axis at which stars of species  $M$  have a bursting interaction with the MBH is then given by

$$a \frac{d\Gamma_M}{da} = (3 - \alpha_M) \frac{C_M N_h}{P(a)} \left( \frac{a}{r_h} \right)^{3-\alpha_M} \left[ \frac{J_{\text{burst}}^2(a) - J_{\text{LSO}}^2}{J_c^2(a)} \right], \quad (8)$$

where  $P(a) = 2\pi(a^3/GM_\bullet)^{1/2}$  is the period. For MS stars, the term  $J_{\text{LSO}}^2$  in equation (8) should be replaced with the tidal loss-cone,  $J_t^2 = 2GM_\bullet r_t$ . Here  $r_t = (M_\bullet/M_\star)^{1/3} R_\star$  is the tidal radius, where a star is disrupted by the tidal force of the MBH.

Equation (8) gives an analytical estimate of the GW burst rate in our GC, assuming that the distribution function can be approximated as being an isotropic power-law distribution, with different powers for different species. It is only valid within the radius of influence  $r_h$ . Since the event rate is entirely dominated by stars very close to the MBH (see Fig. 3), we neglect contributions to the GW burst rate from stars with  $a > r_h$ . From equation (8) it can be seen that for the relevant values of  $\alpha > 1/2$ , the GW burst rate formally diverges for nearby stars (RHBF06): setting  $J_{\text{LSO}} \rightarrow 0$ , the rate is proportional to  $a d\Gamma_M/da \propto a^{1/2-\alpha}$ .

Finally, we note that RHBF06 make a number of cuts in phase space; either these cuts are made here implicitly, or they do not affect our results.

### 3.2 The stellar distribution in the Galactic Centre

The stellar cluster in the GC has been observed in the infrared in much detail. It has been shown (Alexander 1999; Genzel et al. 2003; Alexander 2005) that the stars in the GC are distributed in a cusp with profile  $\rho \propto r^{-1.4}$  consistent with the predictions by Bahcall & Wolf (1976, 1977), although it is important to note that only the most luminous stars can be observed, and that the observations are therefore strongly biased. The stellar population at 1–100 pc is consistent with a model of continuous star formation (Serabyn & Morris 1996; Figer et al. 2004). Within the radius of influence  $r_h = 2$  pc, Genzel et al. (2003) finds that there is a total mass  $M_{\text{tot}} = 1.7 \times 10^6 M_\odot$  in stars.

Not much is known observationally about the inner  $\sim 0.01$  pc of the GC. There are a number of B-stars (known as the ‘S cluster’) at that distance, which provide a challenge for star formation theories, but it is not known whether these stars are representative for the dimmer stars present there: it is more likely that they are the result of tidal binary disruptions by the MBH (Gould & Quillen 2003; Perets, Hopman & Alexander 2007). The S-stars can also be used to probe the enclosed dark mass. This is how the total mass of the MBH,  $M_\bullet = 3.6 \times 10^6 M_\odot$  (Eisenhauer et al. 2005) can be determined, but in principle the orbits of the S-stars can be used to constrain the nature of the extended mass by looking for deviations of Keplerian motion: if, for example, a cluster of stellar BHs is present, the orbits of the S-stars should precess. To date, searches for deviations from Keplerian motion do not lead to relevant constraints (Mouawad et al. 2005).

By lack of direct observations of the stellar content of the inner region of the Galaxy, we resort to theoretical models for mass segregation. Such models were recently made by Freitag et al. (2006) and Hopman & Alexander (2006b), and show that stellar BHs have a much steeper cusp than the other species.

We consider four distinct species of stars: MS stars, WDs, neutron stars (NSs) and stellar BHs, with  $M_{\text{MS}} = 0.5 M_\odot$ ,  $M_{\text{WD}} = 0.6 M_\odot$ ,

$M_{\text{NS}} = 1.4 M_\odot$ ,  $M_{\text{BH}} = 10 M_\odot$ . We assume that the enclosed number of MS stars within the radius of influence at  $r_h = 2$  pc is  $N_h = 3.4 \times 10^6$ , and that the number of compact remnants are equal to that resulting from Fokker–Planck calculations by Hopman & Alexander (2006a), who found  $C_{\text{MS}} = 1$ ,  $C_{\text{WD}} = 0.14$ ,  $C_{\text{NS}} = 9 \times 10^{-3}$ ,  $C_{\text{BH}} = 6 \times 10^{-3}$ . These are the number fractions of all stars within 1 pc, after mass segregation; the fractions of the unbound stars are approximately  $C_{\text{MS}} = 1$ ,  $C_{\text{WD}} = 0.1$ ,  $C_{\text{NS}} = 0.01$ ,  $C_{\text{BH}} = 10^{-3}$ , corresponding to a model of continuous star formation (e.g. Alexander 2005). We note that we ignore the effect of kicks for NSs. Since we show that NSs are unlikely to be observed as GW bursts, this does not significantly change our conclusions. For the slopes, Hopman & Alexander (2006b) found  $\alpha_{\text{MS}} = 1.4$ ,  $\alpha_{\text{WD}} = 1.4$ ,  $\alpha_{\text{NS}} = 1.5$ ,  $\alpha_{\text{BH}} = 2$ . These slopes are all quite different from those assumed by RHBF06, who assumed  $\alpha = 1.75$  for all species.

### 3.3 The inner region of the stellar cusp

The rate of GW bursts is dominated by stars very close to the MBH. It is therefore important to estimate to which distance the cusp continues. Here we consider a number of processes that can determine the inner edge of the cusp.

#### 3.3.1 Finite number effects

Current models of stellar systems near MBHs rely mainly on statistical approaches such as Fokker–Planck methods;  $N$ -body simulations can only be performed for small systems with intermediate-mass black holes (IMBHs) of masses  $M_\bullet \sim 10^3 M_\odot$  (Baumgardt, Makino & Ebisuzaki 2004a,b; Preto et al. 2004). In particular, the Bahcall & Wolf (1976, 1977) solutions which first predicted the slope of the stellar cusp, can in principle extend to any inner radius if there is no physical mechanism that provides a cut-off (such as stellar collisions or tidal disruption). In reality, there is only a finite number of stars; this implies that even if no inner cut-off of the cusp is provided by a physical mechanism that destroys the stars, there is an inner radius beyond which no stars are expected. Statistically, the cusp runs out at<sup>1</sup>

$$r_{1,M} = (C_M N_h)^{-1/(3-\alpha_M)} r_h. \quad (9)$$

Using  $r_h = 2$  pc, this gives for the favoured model  $r_{1,\text{MS}} = 2 \times 10^{-4}$  pc,  $r_{1,\text{WD}} = 6 \times 10^{-4}$  pc,  $r_{1,\text{NS}} = 2 \times 10^{-3}$  pc and  $r_{1,\text{BH}} = 1 \times 10^{-4}$  pc.

It is highly unlikely that there is a star in the GC with semimajor axis  $a \ll r_{1,M}$  during a few years of observations. We therefore neglect in our analytical estimate contributions from such rare cases. However, we do explore this possibility in the Monte Carlo samplings presented in Section 4. These samplings confirm that the probability of the presence of a star within  $r_{1,M}$  is low. Even though such a star would contribute a large number of bursts, it is unlikely to be of importance in the GC.

#### 3.3.2 Hydrodynamical collisions

Close to an MBH, the number density and velocity dispersions become very large, and stars will collide within their lifetimes (Frank & Rees 1976; Cohn & Kulsrud 1978; Murphy, Cohn & Durisen

<sup>1</sup> This expression is also given in Hopman & Alexander (2006a), but with an error in the sign of the exponent.

1991). The rate  $\Gamma_{\text{coll}}$  at which stars with radius  $R_\star$  at a distance  $r_h$  from the MBH have grazing collisions can be estimated as

$$\Gamma_{\text{coll}} = nv\Sigma = \frac{3 - \alpha}{4\pi} \frac{N_h}{r_h^3} \left(\frac{r}{r_h}\right)^{-\alpha} \left(\frac{GM_\bullet}{r}\right)^{1/2} \pi R_\star^2, \quad (10)$$

where  $\Sigma = \pi R_\star^2$  is the cross-section for a grazing collision, when the relative velocity is significantly larger than the escape velocity from the surface of the star.

Studies by (Freitag & Benz 2002, 2005) show a single grazing collision is unlikely to disrupt a star, but that rather  $N_{\text{coll}} \sim 20\text{--}30$  collisions are required for disruption (Freitag et al. 2006). This implies that stars are disrupted by collisions within a Hubble time if their distance from the MBH is smaller than

$$r_{\text{coll}} = 3 \times 10^{-3} \text{ pc} \left(\frac{N_{\text{coll}}}{30}\right)^{1/2} \left(\frac{R_\star}{R_\odot}\right), \quad (11)$$

where it was assumed that  $\alpha = 3/2$ , as is approximately the case for MS stars.

Although this estimate is clearly not very precise, it is unlikely that the cusp for MS stars continues much closer to the MBH than  $r_{\text{coll}}$ , since collisions become very frequent and with higher impact velocities. For the preferred model we assume  $r_{\text{coll}}$  as the inner cut-off of the cusp for MS stars. We do not consider collisions between other stellar species.

### 3.3.3 Gravitational wave inspiral

GW emission plays a double role: on one hand the GWs can be detected by *LISA*, but on the other they also change the dynamics close to the MBH, since the star emitting the GW loses orbital energy, and spirals in. Close to the MBH, stars spiral in faster than they are replenished by other stars. This region of phase space is therefore typically empty, because any bursting star would be quickly accreted by the MBH.

If  $r_p \ll a$ , the inspiral time is approximately given by (Peters 1964)

$$t_0(r_p, a) = \frac{2\pi\sqrt{GM_\bullet a}}{\Delta E_{\text{GW}}(r_p, a)}, \quad (12)$$

where

$$\Delta E_{\text{GW}}(r_p, a) = \frac{2\pi}{5\sqrt{2}} f(e) \frac{M_\star c^2}{M_\bullet} \left(\frac{r_p}{r_s}\right)^{-7/2} \quad (13)$$

and  $f(e) \approx 2.2$ . If this time-scale at  $r_p = r_p^{\text{burst}}$  is much smaller than the time-scale  $t_J(r_p, a) \sim (r_p/a)t_r$  for two-body scattering to change the angular momentum by an order of unity, stars with  $r_p < r_p^{\text{burst}}$  spiral in much faster than they are replenished. Solving  $t_J(r_p^{\text{burst}}, a) = t_0(r_p^{\text{burst}}, a)$  for  $a$  gives an inner cut-off

$$a_{\text{GW}} = 1.9 \times 10^{-4} \text{ pc} \left(\frac{m}{M_\odot}\right)^{2/13} \left(\frac{d}{8 \text{ kpc}}\right)^{20/39} \left(\frac{\rho}{5}\right)^{20/39}, \quad (14)$$

where a relaxation time of  $t_r = 10^9$  yr was assumed.

### 3.3.4 Kicks out of the cusp

An important assumption that is routinely made in stellar dynamics is that the rate at which stars exchange energy and angular momentum is dominated by small-angle encounters (e.g. Chandrasekhar 1943; Binney & Tremaine 1987). This is justified by comparing the large-angle scattering time-scale,  $t_{\text{LA}} \approx [nv(GM_\star/v^2)^2]^{-1}$ , to

the relaxation time  $t_r$ . The time-scale for large-angle scattering by a single strong encounter is larger than the relaxation time (where many small encounters add up to a large angle) by a factor  $t_{\text{LA}}/t_r \sim \ln \Lambda$ , where  $\ln \Lambda$  is the Coulomb logarithm; close to an MBH,  $\Lambda \sim M_\bullet/M_\star$  (Bahcall & Wolf 1976).

In spite of this, large-angle scattering may play an important role in the ejection of stars out of the cusp (Lin & Tremaine 1980; Baumgardt et al. 2004a). Whether the rate of ejections out of the cusp is larger than the rate at which stars are swallowed by the MBH may depend on the size of the system: Lin & Tremaine (1980) and Baumgardt et al. (2004a) find that the ejection rate is larger for IMBHs of  $M_\bullet \sim 10^3 M_\odot$ , but the swallow rate is much higher for MBHs (Freitag et al. 2006).

Even if the ejection rate is larger than the merger rate, in all cases the rate at which stars are replenished by diffusion in energy space is larger than the ejection rate by a factor  $\ln \Lambda$  (Bahcall & Wolf 1977; Lin & Tremaine 1980). Ejections can therefore never deplete the inner region of the cusp, and need not be considered for the purposes of this paper. We note that Baumgardt et al. (2004a) found that all stellar BHs are ejected, but this cannot happen in a galactic nucleus where these objects are constantly replenished by mass segregation from larger radii.

### 3.3.5 The role of the loss-cone

We assume an isotropic velocity distribution for the stars, leading to the DF  $n(a, J^2)$  given in equation (7). We do not consider stars in the region  $J < J_{\text{LSO}}$ , which is the ‘loss-cone’; loss-cone theory shows that so close to the MBH, there are no stars in this region in phase space, because any star will be immediately removed (Lightman & Shapiro 1977; Cohn & Kulsrud 1978).

In reality, there will be a smooth transition from the empty region of the loss-cone to the region far away from the loss-cone (large angular momenta). Lightman & Shapiro (1977) find that close to the loss-cone, there is a logarithmic depletion of stars. Taking this factor into account leads to a suppression of the GW burst rates by a factor of the order of  $\sim 3$  compared to the results we present here. On the other hand, resonant relaxation (Rauch & Tremaine 1996) may replenish some stars to this region (Rauch & Ingalls 1998), although the effect will be not very large due to general relativistic precession, which destroys the resonant relaxation.

In this paper we do not consider any modification of the DF by the presence of the loss-cone, but note that this approach may be somewhat optimistic.

## 3.4 Main model

To summarize, the method to compute the GW burst rate is as follows. Four species of stars (MS, WD, NS, BH) are considered, all with their own mass  $m/M_\odot = (0.5, 0.6, 1.4, 10)$ , number normalization  $C_M = (1, 0.14, 0.009, 6 \times 10^{-3})$  at  $r_h = 2$  pc, slope  $\alpha_M$ , inner radius  $r_{\text{in},M}$  and loss-cone  $J_{\text{lc}} = \max(J_{\text{LSO}}, J_t)$ . The total number of MS stars ( $C_M = 1$ ) within the cusp is  $N_h = 3.4 \times 10^6$ . These values are used in equation (8); the rate  $\Gamma_M$  is then integrated to find the total GW burst rate for each species.

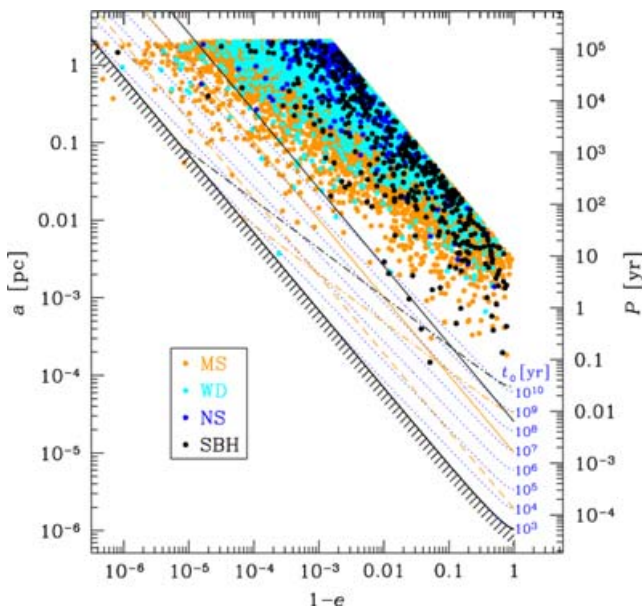
For the model which is regarded to reflect the stellar population in the GC best, the following values are assumed. For the slopes,  $\alpha_M = (1.4, 1.4, 1.5, 2.0)$ ; for the inner radius,  $r_{\text{in}} = \max(r_{\text{coll}}, r_1, a_{\text{GW}})$ , we found  $r_{\text{in},\text{MS}} = 3 \times 10^{-3}$  pc (collisions),  $r_{\text{in},\text{WD}} = 6 \times 10^{-4}$  pc,  $r_{\text{in},\text{NS}} = 2 \times 10^{-3}$  pc (finite number effects) and  $r_{\text{in},\text{BH}} = 3 \times 10^{-4}$  pc (GW inspiral).

We consider the parameters assumed here to be representative for the GC. There is considerable uncertainty in some of the assumptions, in particular in the abundance and masses (and possible distribution of masses) of the stellar BHs. However, it was shown in dynamical simulations by Freitag et al. (2006) that after 10 Gyr the number of stellar BHs close to the MBH was comparable for several different initial mass functions. Results for other choices of the parameters can easily be evaluated using the expressions in the previous sections.

#### 4 MONTE CARLO REALIZATIONS OF STELLAR CUSPS

The analytical method described in the previous section is useful to obtain an estimate of the average burst rate. However, it discards rare events where a single star comes very close to the MBH and has a large number of bursts per year. It also does not give information on the distribution of the burst rate. In order to obtain this information we complemented our analytical estimate by a Monte Carlo approach, in which we produce a large number of realizations of the models discussed in the previous section. This gives us the cumulative probability  $P(>\Gamma)$  that the event rate is higher than  $\Gamma$ . In the Monte Carlo samplings, we do not need to explicitly include a cut-off at small radii to account for statistical depletion. Stars with  $(1 - e)t_r > t_0(e, a)$  are discarded (see equation 3.3.3). Other cuts are identical to those made in the analytical approach.

One example of a realization of the stellar cusp is shown in Fig. 1.



**Figure 1.** Realization of the main model for the stellar cusp, with  $1 - e$  on the horizontal axis and the semimajor axis on the vertical axis (or period on the right-hand vertical axis). The lightest dots represent MS stars, then WDs, NSs, and finally the black dots represent BHs, with numbers and slopes as described in the main model. The black line with hatching demarcates the last stable orbit. The dotted blue lines give the GW inspiralling time (equation 12) for the MS stars. The solid grey line indicates the ‘burst region’ for MS stars (with  $S/N = 5$ ). The solid black line above it is for stellar BHs. The dashed line is the tidal disruption radius for MS stars. The grey dot-dashed line shows  $t_0 = (1 - e)t_r$  for MS stars while the black one is for stellar BHs. Below these lines, depletion by GW inspiral should occur. The region with  $P < 1$  yr, where most bursting stars should be, is almost entirely depleted due to the finite number effect.

## 5 RESULTS

A number of different possibilities were considered for the slopes and inner cut-offs of the respective stellar populations. The resulting GW burst rates for these models are summarized in Table 1.

For our main model, we find that GW bursts are unlikely to be observed in the GC for MS stars ( $\Gamma_{\text{MS}} \sim 0.1 \text{ yr}^{-1}$ ), for WDs ( $\Gamma_{\text{WD}} \sim 0.1 \text{ yr}^{-1}$ ) and for NSs ( $\Gamma_{\text{NS}} \sim 0.004 \text{ yr}^{-1}$ ). Our burst rate for MSs is much lower than the  $\Gamma_{\text{MS}} \sim 12 \text{ yr}^{-1}$  rate estimated by RHBF06; the main reason for the difference is the cut-off due to collisions. Our rates for WDs and NSs are also lower than those found by RHBF06 (who estimated  $\Gamma_{\text{WD}} \sim 3 \text{ yr}^{-1}$  and  $\Gamma_{\text{NS}} \sim 0.1 \text{ yr}^{-1}$ ); here the difference is probably caused mainly by the different density profile, and the fact that the cusp runs out of stars at small radii from the MBH. On the other hand, we find a higher rate of BH bursts, ( $\Gamma_{\text{BH}} \sim 1 \text{ yr}^{-1}$ ), which is the result of the steeper density profile we assumed, caused by mass segregation (Freitag et al. 2006; Hopman & Alexander 2006b). The inner radius for BHs was determined by GW inspiral in this case (Section 3.3.3).

The cumulative probability to detect more than a certain number of bursts per year can be determined with Monte Carlo sampling (Section 4). It consists of several factors, including the probability that in spite of the finite number effect a star has a very short period in a certain realization. In this latter case there is a large number of correlated bursts, so that the distribution is not Poissonian. We show the results for the main model in Fig. 2. The average rates are in good agreement with our analytical model. Smaller differences are that for WDs and NSs, the Monte Carlo rates are slightly higher because of rare events excluded in the analytical model, while for BHs the rates are slightly lower, due to a small difference in the criterion for GW inspiral. From the figure it can be confirmed that the probability to observe even one single burst for MSs, WDs and NSs is negligible, but there is some chance to observe several GW bursts from BHs. The probability that the rate of observed BH bursts per year exceeds 1 is  $P(>1 \text{ yr}^{-1}) \approx 20$  per cent. For illustration purposes, we show in Fig. 1 an example of a realization of the main model.

To probe the sensitivity of the GW burst rate to the assumptions, a number of other possibilities are considered explicitly. *We stress that these models lack in realism; we consider them with the purpose of probing how sensitive our results are to the assumptions made.*

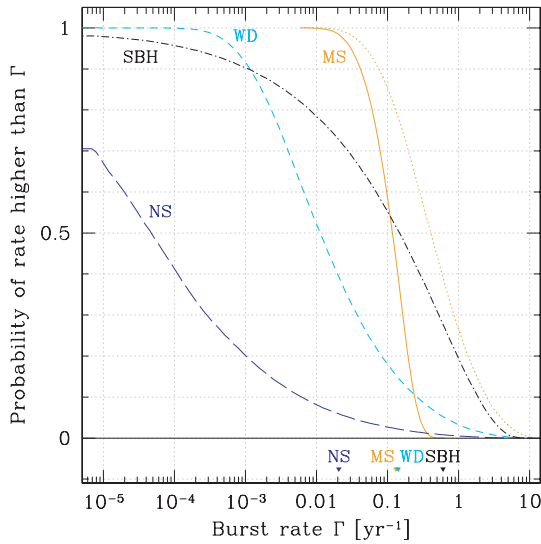
First, consider the possibility that there is mass segregation, but an inner cut-off of only  $a_{\text{min}} = 3 \times 10^{-5}$  pc for all stars (this is approximately where bursting sources become continuous sources, see equation 5 and RHBF06). This increases the rate considerably for all species. The GW burst rate is plotted in Fig. 3. From this figure it can also be seen what the event rates for the main model are, by considering the appropriate cut-off for each species.

Alternatively, we consider the case that there is an inner cut-off equal for all species similar to that of the main model, but that the slope is  $\alpha_M = 1.75$  for all species, as would be the case for a relaxed, single-mass population.<sup>2</sup> We used a normalization ( $C_{\text{MS}} : C_{\text{WD}} : C_{\text{NS}} : C_{\text{BH}} = 1 : 0.1 : 0.01 : 1 \times 10^{-3}$ ) here. This example would present some realism if all stars are of similar mass and in particular if the typical mass of stellar BHs would be of the order of  $M_{\text{BH}} \sim 1 M_{\odot}$ , or if the number of stellar BHs would be much smaller than assumed here (in the latter case the burst rate of the BHs would of course be

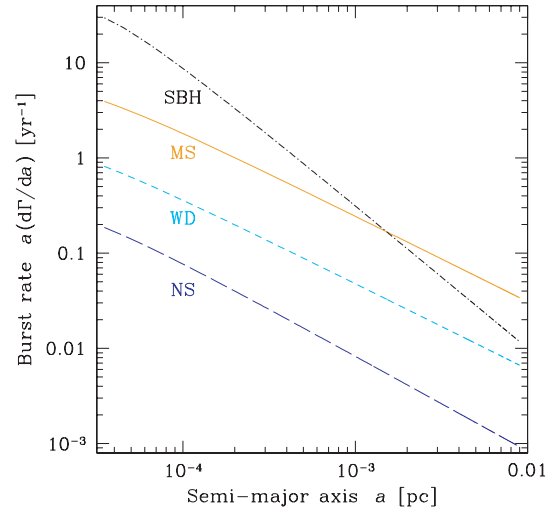
<sup>2</sup> We note, however, that in absence of massive bodies such as stellar BHs, significant relaxational evolution is not to be expected over a Hubble time (Freitag et al. 2006). Without significant relaxation, the density profile of all objects is probably not steeper than  $\rho \propto r^{-1.5}$ .

**Table 1.** Event rates for a number of different stellar species and slopes. For all cases the required S/N for detection was assumed to be  $\rho = 5$ . The first four entries give the favoured model, which accounts for mass segregation according to the results by Hopman & Alexander (2006b), and has an inner cut-off of the cusp due to stellar collisions or finite number effects. The following four entries give the same model, but with equal inner cut-offs  $r_{\text{in}} = 3 \times 10^{-5}$  pc for all stars. The next eight entries are without mass segregation, but with an inner cut-off; this could be appropriate if there are no SBHs (although they do appear in the table), in which case mass segregation would be much less extreme. The slopes presented are  $\alpha = 1.75$  and 1.5. The last four entries also have the same inner cut-off, and an equal slope  $\alpha = 1.75$  as appropriate for a single-mass cusp, similar to what was assumed by RHBF06.

Model	Star	$m$ ( $M_{\odot}$ )	$C_M$	$\alpha_M$	$a_{\text{in}}$ (pc)	Reason for cut-off	$\Gamma_M$ ( $\text{yr}^{-1}$ )
Mass segregation, cut-off (main model)	MS	0.5	1.0	1.4	$3 \times 10^{-3}$	Collisions	0.1
	WD	0.6	0.14	1.4	$6 \times 10^{-4}$	Finite number	0.09
	NS	1.4	0.009	1.5	$2 \times 10^{-3}$	Finite number	$4 \times 10^{-3}$
	SBH	10	$6 \times 10^{-3}$	2.0	$3 \times 10^{-4}$	GW inspiral	1.5
Mass segregation, no cut-off	MS	0.5	1.0	1.4	$3 \times 10^{-5}$	–	6
	WD	0.6	0.14	1.4	$3 \times 10^{-5}$	–	1.2
	NS	1.4	0.009	1.5	$3 \times 10^{-5}$	–	0.2
	SBH	10	$6 \times 10^{-3}$	2.0	$3 \times 10^{-5}$	–	30
No mass segregation, cut-off	MS	0.5	1.0	1.75	$3 \times 10^{-3}$	Collisions	0.5
	WD	0.6	0.1	1.75	$1 \times 10^{-4}$	GW inspiral	4
	NS	1.4	0.01	1.75	$5 \times 10^{-4}$	Finite number	0.1
	SBH	10	$1 \times 10^{-3}$	1.75	$3 \times 10^{-3}$	Finite number	$2 \times 10^{-3}$
No mass segregation, cut-off	MS	0.5	1.0	1.5	$3 \times 10^{-3}$	Collisions	0.16
	WD	0.6	0.1	1.5	$4 \times 10^{-4}$	Finite number	0.17
	NS	1.4	0.01	1.5	$2 \times 10^{-3}$	Finite number	0.005
	SBH	10	$1 \times 10^{-3}$	1.5	$9 \times 10^{-3}$	Finite number	$2 \times 10^{-4}$
No mass segregation, no cut-off	MS	0.5	1.0	1.75	$3 \times 10^{-5}$	–	280
	WD	0.6	0.1	1.75	$3 \times 10^{-5}$	–	42
	NS	1.4	0.01	1.75	$3 \times 10^{-5}$	–	5
	SBH	10	$1 \times 10^{-3}$	1.75	$3 \times 10^{-5}$	–	0.7



**Figure 2.** Distribution of burst rates for 40 000 Monte Carlo realizations. We plot the probability for the burst rate of each stellar species to be larger than a value  $\Gamma$  as a function of  $\Gamma$ . Each cusp realization consists of  $N_h = 3.4 \times 10^6$  particles distributed around an MBH according to the parameters of the main model, such as illustrated in Fig. 1. Stars on plunge orbits or with  $(1 - e)r_r > t_0(r_p, a)$  (see Section 3.3.3) are discarded. We also remove MS stars with periastron distance smaller than the tidal disruption radius  $r_t \simeq 2 \times 10^{-6}$  pc or semimajor axis smaller than the collision radius  $r_{\text{coll}} = 3 \times 10^{-3}$  pc. The dotted line indicate the rate distribution for MS stars if there were no collisional depletion ( $r_{\text{coll}} = 0$ ). The triangles above the horizontal axis indicate the rates averaged over all realizations. Notice that, except for MS stars, they are much higher than the median rates.



**Figure 3.** The rate  $a d\Gamma_M / da$  for the preferred model with  $(\alpha_{\text{MS}}, \alpha_{\text{WD}}, \alpha_{\text{NS}}, \alpha_{\text{BH}}) = (1.4, 1.4, 1.5, 2)$  and  $(C_{\text{MS}}, C_{\text{WD}}, C_{\text{NS}}, C_{\text{BH}}) = (1, 0.14, 0.009, 6 \times 10^{-3})$ . The required S/N for detection was assumed to be  $\rho = 5$ . The lines are extended for the case where there is no inner cut-off to the distribution function. In our main model, the rate does have an inner cut-off, see Table 1.

lower). In this case, the GW burst rate would be dominated by WDs, with a rate of the order of  $\Gamma_{\text{WD}} \sim 4 \text{ yr}^{-1}$ . Interestingly, the cut-offs for this model are determined by different mechanism than for our main model (see Table 1). We also consider the same model, but with all slopes equal to  $\alpha = 1.5$ . This is comparable to the slope of the *observed* population of stars in the GC (Genzel et al. 2003); this population consists of young, massive stars and luminous giants,

and is not necessarily related to the populations that are of interest for our purposes. Since this slope implies that there are even less stars very close to the MBH, the rates are much lower.

Finally, equation (8) was applied to model parameters similar to those assumed by RHBF06, that is, without mass segregation and with a fixed, very small inner cut-off. In this case an event rate is found that is more than an order of magnitude above that found by RHBF06. It is unclear what causes the discrepancy.

## 6 SUMMARY AND DISCUSSION

When stars come very close ( $r_p \lesssim 60r_s$ , see equation 5) to the MBH in our GC, they emit a burst of GWs that could be observable by *LISA* (RHBF06). In this paper an analytical estimate for the burst rate is given. The estimate includes physics that was not considered by RHBF06, in particular mass segregation and processes which determine the inner cut-off of the stellar distribution function. Mass segregation mostly leads to different contributions from different species. However, since the event rate is dominated by stars very near the MBH (equation 8), the inner cut-off leads to a strong suppression of the GW burst rate. We find that only stellar BHs have a reasonable chance of being observed as bursting sources, with a rate of the order of  $\Gamma \sim 1 \text{ yr}^{-1}$  for  $\rho = 5$ .

The stellar distribution function in the inner 0.01 pc is not known in the GC, and the results presented here rely on theoretical estimates, rather than on observations. The role of collisions on the inner structure of the cusp is still poorly known, and if the inner cut-off would be considerably smaller than assumed here, the GW burst rate for MS stars grows substantially. Observation of a number of GW bursts from the GC would therefore have implications for our understanding of stellar dynamics near MBHs. The observation of a GW burst would probably allow one to constrain the masses of the system. In our models, we find that stellar BHs are the most likely candidates to be bursting sources. However, if the bursting source is a WD, then this would imply that either stellar BHs have masses much lower than  $10 M_\odot$ , or that their number is much smaller than assumed here; in both cases the distribution of WDs would be steeper than we assumed, and our model with cut-off, but without mass segregation, indicates that several WD bursts per year are then to be expected. Interestingly, similar conclusions would apply for inspiral sources (Hopman & Alexander 2006b).

Using stellar dynamics simulations, Freitag (2003) suggested that, at any given time there are  $\sim 1\text{--}3$  *continuous* GW sources at the GC (i.e. EMRIs), namely MS stars with a mass of  $\sim 0.05\text{--}0.1 M_\odot$  on orbits with  $P \lesssim 3 \times 10^4$  s. This result would imply a burst rate much higher than estimated here. We note that a large population of low-mass MS stars would lead to a slightly higher burst rate because of the larger number of stars and the weaker depletion by GW emission. However, the EMRI rates obtained by Freitag (2003) seem to have been overestimated, due to the approximate treatment of the condition for GW-driven inspiral relying on a noisy particle-based estimate of the relaxation time. Furthermore, in that work, once it had reached the GW-dominated regime the possibility for a MS star to be destroyed by collisions was neglected.

We stress that a star on route to become an EMRI is unlikely to be a bursting source: the event rate at which EMRIs are created is of the order of  $\Gamma_{\text{EMRI}} \sim 0.1 \text{ Myr}^{-1}$ , while the time which a typical future EMRI spends at orbits with periods less than 1 yr is  $t_i \sim 0.05 \text{ Myr}$  (Hopman & Alexander 2006b), implying that the probability of observing such a source in the GC is of the order of  $\sim t_i \Gamma_{\text{EMRI}} \sim 5 \times 10^{-3}$ . This confirms our assumption that the inner regions of the cusp are depleted in presence of GW energy losses (Section 3.3.3).

Bursts of GWs from stars passing close to extragalactic MBHs are a potential source of noise for *LISA*. An estimate of the contribution to *LISA*'s noise budget is out of the scope of this paper, and will be considered elsewhere.

RHBF06 also considered the possibility of observing GW bursts from the Virgo cluster, and estimated that only stellar BHs could be observed as bursting sources, with of the order of three bursts per year. We note that our higher rate of bursting BHs in the centre of our Galaxy than that found by RHBF06 does *not* imply that we also predict a higher rate of bursts from Virgo: for fixed S/N, a smaller periape is required in Virgo, which in turn implies a larger inner cut-off of the density profile (see equation 14). Taking this into account, we find that the bursting rate in Virgo is only of the order of  $\sim 10^{-4} \text{ yr}^{-1}$  per galaxy. Since there are about 2100 galaxies in the Virgo cluster (Binggeli, Sandage & Tammann 1985), and only a fraction of these have MBHs with relevant masses, no GW bursts are expected to be detected in the Virgo cluster.

Finally, we note that the farthest distance to which non-repeating, bursting  $10 M_\odot$  stellar BHs can still be detected is  $\sim 100 \text{ Mpc}$ . The rate at that distance is  $\sim 10^{-6} \text{ yr}^{-1}$ . Since the local MBH density of  $\sim 10^6 M_\odot$  MBHs is  $\sim 10^{-2} \text{ Mpc}^{-3}$  (Aller & Richstone 2002), this yields an annual detection probability of 1 per cent.

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