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The LISA zero-signal solution

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Abstract
We derive a time-delay interferometric (TDI) combination that has zero-response to a gravitational wave signal. This combination, which we have called the *zero-signal solution*, is a two-parameter family of linear combinations of the generators of the TDI space that has null gravitational wave response when its two parameters coincide with the values of the angles of the source location in the sky. Remarkably, the zero-signal solution does not rely on any assumptions about the gravitational waveform, and in fact it works for waveforms of any kind.

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1. Introduction

Earth-based interferometer detectors operate in a frequency band where the wavelength of the gravitational wave (GW) signal they are trying to detect is much longer than their arm length. LISA, in contrast, will *not* operate in the ‘long-wavelength limit’ over much of its frequency band. For this reason, time delays in the response of the instrument to the waves, and travel times along beams in the instrument, cannot be ignored and must be allowed for when deriving the correct expression for the detector response.

LISA will rely on phase measurements made by its 12 Doppler readouts in order to compensate for the inequality of its arms and suppress the laser frequency fluctuations below the level identified by the secondary noise sources. This is done by properly time shifting and linearly combining digitally the various phase measurements, resulting in data combinations that show different couplings to gravitational radiation and to the remaining system noises. This data processing technique has been called time-delay interferometry (TDI), and it has been shown that the functional space it generates can be obtained by properly time-shifting and linearly combining four generators[3] $(\alpha, \beta, \gamma, \zeta)$. In particular, there exists a combination of the four generators that, for a given waveform and source location in the sky, achieves a maximum signal-to-noise ratio [1].

Here we further elaborate on the geometrical properties of the four generators' responses. We find that there exists a two-parameter family of combinations of $(\alpha, \beta, \gamma, \zeta)$ that has null response to an observed gravitational wave signal. In the case of gravitational wave bursts, or sinusoidal signals observed over a time scale short enough that the amplitude and frequency modulation induced in the received signal by the LISA motion around the Sun are negligible, this new TDI combination shows a sharp null when its two parameters coincide with the values of the angles describing the source location in the sky, or its mirror image with respect to the LISA plane. This combination, which we call the *zero-signal solution* (ZSS) [2], does not rely on any assumptions about the gravitational waveform, and in fact it works for gravitational waveforms of any kind. Although its ability to identify these two points in the sky degrades at lower frequencies, it should be regarded as an improvement over the symmetric Sagnac combination, ζ [3], since it could be used for discriminating gravitational wave signals from spurious fluctuations generated by instrumental noise in the entire LISA frequency band. The ZSS is analogous to the data analysis method introduced by Gürsel and Tinto in the context of networks of Earth-based, wide-band, interferometric gravitational wave detectors observing in coincidence with a GW burst [4], and it should be regarded as an application of the Gürsel and Tinto method to the LISA data.

The ability of the ZSS to identify the source location in the sky follows from the fact that the time spent by the gravitational wavefront in propagating across the LISA constellation enters the gravitational wave responses of the four generators $(\alpha, \beta, \gamma, \zeta)$. The two independent travel times spent by the wave to propagate from one spacecraft to the other two define the 'six-pulse' structure in the responses of the four generators. Since the signal contribution to the ZSS combination becomes null at the correct values of these two time-delays, and the antenna pattern of the ZSS is symmetric with respect to the LISA plane, it follows that in general there are two possible points in the sky where the signal must have originated. If the duration of the signal is such that the effects of the motion of the LISA array around the Sun can be disregarded (as in the case of bursts), then the two-fold degeneracy cannot be removed. However, if the duration of the signal is such that the effects of the motion of LISA around the solar system barycentre can be measured (as in the case of signals from binary systems), then the source location can be uniquely identified. As LISA moves around the Sun, the point in the sky where the source is located will remain fixed, while the other will move in such a way to remain its mirror image with respect to the LISA plane. By constructing the ZSS with different stretches of data, the unique source location can be identified.

The ZSS will be particularly useful when observing gravitational wave bursts, or continuous signals that are strong enough to be observed by integrating the data over time scales shorter than the time scale during which the phase and amplitude modulations of the signal become measurable. In the latter case, the ZSS should be regarded as the first step of a hierarchical procedure for measuring the parameters of a continuous signal. The ZSS could be particularly useful when observing, for instance, gravitational radiation emitted during the inspiral of two super-massive blackholes, or one smaller object into a super-massive hole. The ability of the ZSS to identify the location of the binary system in the sky, without relying on knowledge of the signal waveform, might reduce significantly the number of templates needed in order to study waveforms from such astrophysical systems [5].

[3] On TDI, see [6, 7] and references therein.

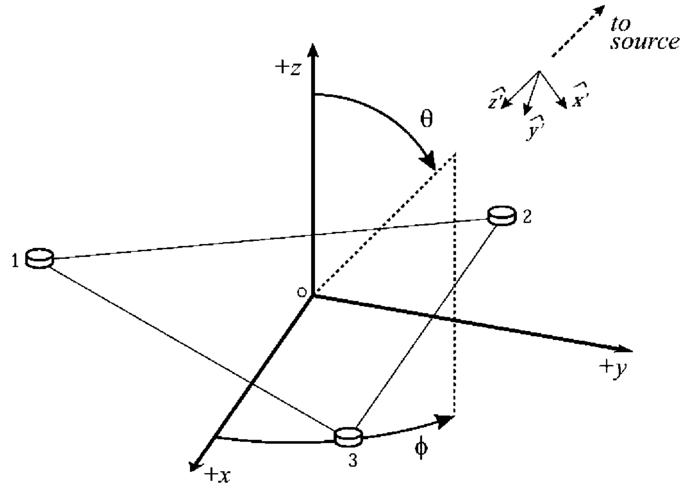

Figure 1. The array and the wave coordinates. The x, y and z axes are the LISA coordinate axes with the origin at the point O, and the array lies in the x, y plane. The x', y', z' axes are the coordinate axes for the incoming gravitational wave. The z' axis is parallel to the direction of propagation of the wave. The x', y' axes are those with respect to which $h_+(t)$ and $h_\times(t)$ are defined. The angles θ, ϕ describe the location of the source in the sky.

2. Derivation of the ZSS solution

Let us consider the *second generation* TDI Sagnac observables $(\alpha_1, \alpha_2, \alpha_3)$. The expressions for the gravitational wave signal and the secondary noise sources entering α_1, for instance, will in general be different from those entering α, the corresponding Sagnac observable derived under the assumption of a stationary LISA array. However, since the gravitational wave signal and the secondary noises in LISA are so much smaller than the laser noises, and the rotation and systematic velocities in LISA are so intrinsically small, it follows that the flexing and Sagnac effects on them can be ignored [7]. This implies the following relationship between the signal and secondary noises in α_1 and those entering the combination α

$$\alpha_1(t) \simeq \alpha(t) - \alpha(t - L_1 - L_2 - L_3), \tag{1}$$

where L_i, $i = 1, 2, 3$ are the unequal-arm lengths of the stationary LISA array. Equation (1) implies that any data analysis procedure and algorithm that will be implemented for the α_i combinations can actually be derived by considering the corresponding combinations valid for a stationary array. For this reason, from now on we will focus our attention on the gravitational wave responses of the observables $(\alpha, \beta, \gamma, \zeta)$. In the reference frame co-moving with LISA and centred at the point O that is equidistant from the three spacecraft (see figure 1), the Doppler response of α to a gravitational wave signal is given by the following expression:

$$\begin{aligned}
\alpha^{\mathrm{gw}} = {} & \left[1 - \frac{l}{L_1}(\mu_2 - \mu_3)\right](\Psi_1(t - \mu_2 l - L_1 - L_2) - \Psi_1(t - \mu_3 l - L_2)) \\
& - \left[1 + \frac{l}{L_1}(\mu_2 - \mu_3)\right](\Psi_1(t - \mu_3 l - L_1 - L_3) - \Psi_1(t - \mu_2 l - L_3)) \\
& + \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right](\Psi_2(t - \mu_3 l - L_2) - \Psi_2(t - \mu_1 l))
\end{aligned}$$

$$
\begin{aligned}
&-\left[1+\frac{l}{L_2}(\mu_3-\mu_1)\right](\Psi_2(t-\mu_1 l-L_2-L_1-L_3)-\Psi_2(t-\mu_3 l-L_1-L_3))\\
&+\left[1-\frac{l}{L_3}(\mu_1-\mu_2)\right](\Psi_3(t-\mu_1 l-L_3-L_1-L_2)-\Psi_3(t-\mu_2 l-L_1-L_2))\\
&-\left[1+\frac{l}{L_3}(\mu_1-\mu_2)\right](\Psi_3(t-\mu_2 l-L_3)-\Psi_3(t-\mu_1 l)), \qquad (2)
\end{aligned}
$$

where $\mu_i = \hat{k}\cdot\hat{p}_i$, $\hat{p}_i$, $i = 1, 2, 3$, is the unit vector pointing from O to spacecraft i and $\hat{k}$ is the unit propagation vector of the wave. The function Ψ_i is equal to

$$
\Psi_i(t) = \frac{1}{2}\frac{\hat{n}_i\cdot\mathbf{h}(t)\cdot\hat{n}_i}{1-(\hat{k}\cdot\hat{n}_i)^2}, \qquad (3)
$$

where $\mathbf{h}(t)$ is the first-order spatial metric perturbation at the point O. Note that $L_1\hat{k}\cdot\hat{n}_1 = l(\mu_2-\mu_3)$, and so forth by cyclic permutation of the indices.

Equation (2) for $\alpha^{\rm gw}$ shows a 'six-pulse response' to gravitational radiation. In other words, a δ-function gravitational wave signal produces six distinct pulses in (α, β, γ), which are located with relative times depending on the arrival direction of the wave and the detector configuration.

In order to derive the expression for the ZSS, let us consider in the Fourier domain the following linear combination, $\widetilde{\eta}$, of the Fourier transforms of (α, β, γ):

$$
\widetilde{\eta}(f) \equiv a_1(f,\vec{\lambda})\widetilde{\alpha}(f) + a_2(f,\vec{\lambda})\widetilde{\beta}(f) + a_3(f,\vec{\lambda})\widetilde{\gamma}(f), \qquad (4)
$$

where the $\{a_i(f,\vec{\lambda})\}_{i=1}^3$ are arbitrary complex functions of the Fourier frequency f, and of a vector $\vec{\lambda}$ containing parameters characterizing the gravitational wave signal (source location in the sky, waveform parameters, etc). Note that this combination represents an arbitrary element of the TDI space since the Fourier transform of ζ is itself a linear combination of the other three Sagnac combinations (α, β, γ) [2]. Our goal is to determine the functions $\{a_i\}_{i=1}^3$, in such a way that the gravitational wave signal in $\widetilde{\eta}(f)$, i.e. $\widetilde{\eta}^{\rm gw}(f)$, is identically zero. Since the expressions for $\alpha^{\rm gw}$, $\beta^{\rm gw}$ and $\gamma^{\rm gw}$ can be rewritten in the following form (see [2]):

$$
\widetilde{\alpha}^{\rm gw}(f) = \alpha_+(f,\theta_s,\phi_s)\widetilde{h}_+(f) + \alpha_\times(f,\theta_s,\phi_s)\widetilde{h}_\times(f), \qquad (5)
$$

$$
\widetilde{\beta}^{\rm gw}(f) = \beta_+(f,\theta_s,\phi_s)\widetilde{h}_+(f) + \beta_\times(f,\theta_s,\phi_s)\widetilde{h}_\times(f), \qquad (6)
$$

$$
\widetilde{\gamma}^{\rm gw}(f) = \gamma_+(f,\theta_s,\phi_s)\widetilde{h}_+(f) + \gamma_\times(f,\theta_s,\phi_s)\widetilde{h}_\times(f), \qquad (7)
$$

where (θ_s, ϕ_s) are the two angles describing the location of the source in the sky with respect to a LISA coordinate frame (see figure 1), and the functions $\alpha_+, \alpha_\times, \beta_+, \beta_\times, \gamma_+, \gamma_\times$ are generalized pattern functions [2], it follows that $\widetilde{\eta}^{\rm gw}$ can be rewritten in the following form:

$$
\begin{aligned}
\widetilde{\eta}^{\rm gw}(f) = {}&[a_1\alpha_+(f,\theta_s,\phi_s) + a_2\beta_+(f,\theta_s,\phi_s) + a_3\gamma_+(f,\theta_s,\phi_s)]\widetilde{h}_+(f)\\
&+[a_1\alpha_\times(f,\theta_s,\phi_s) + a_2\beta_\times(f,\theta_s,\phi_s) + a_3\gamma_\times(f,\theta_s,\phi_s)]\widetilde{h}_\times(f), \qquad (8)
\end{aligned}
$$

after substituting equations (5)–(7) into equation (4). The wave's two independent amplitudes, $h_+, h_\times$, are referred to the wave axes (x', y') which, without loss of generality, can be assumed to be oriented in such a way that the x'-axis lies parallel to the LISA plane (the x–y plane). In order to have $\widetilde{\eta}^{\rm gw} = 0$ for any arbitrary pairs of wave amplitudes $h_+, h_\times$, equation (8) implies that the following homogeneous linear system of two equations in three unknowns must be satisfied

$$
a_1\alpha_+(f,\theta_s,\phi_s) + a_2\beta_+(f,\theta_s,\phi_s) + a_3\gamma_+(f,\theta_s,\phi_s) = 0 \qquad (9)
$$

$$a_1\alpha_\times(f,\theta_s,\phi_s)+a_2\beta_\times(f,\theta_s,\phi_s)+a_3\gamma_\times(f,\theta_s,\phi_s)=0. \tag{10}$$

Since the rank of the matrix associated with the linear system above is in general equal to 2, it is easy to derive the expressions for a_1, a_2, a_3 that give zero-response to a gravitational wave signal at the source location

$$a_1(f,\theta_s,\phi_s)=\beta_+(f,\theta_s,\phi_s)\gamma_\times(f,\theta_s,\phi_s)-\beta_\times(f,\theta_s,\phi_s)\gamma_+(f,\theta_s,\phi_s), \tag{11}$$

$$a_2(f,\theta_s,\phi_s)=\gamma_+(f,\theta_s,\phi_s)\alpha_\times(f,\theta_s,\phi_s)-\gamma_\times(f,\theta_s,\phi_s)\alpha_+(f,\theta_s,\phi_s), \tag{12}$$

$$a_3(f,\theta_s,\phi_s)=\alpha_+(f,\theta_s,\phi_s)\beta_\times(f,\theta_s,\phi_s)-\alpha_\times(f,\theta_s,\phi_s)\beta_+(f,\theta_s,\phi_s). \tag{13}$$

Equations (11)–(13) imply that the following linear combination of the Fourier transforms of the three generators (α, β, γ):

$$\begin{aligned}\widetilde{\eta} \equiv {} & [\beta_+(f,\theta,\phi)\gamma_\times(f,\theta,\phi)-\beta_\times(f,\theta,\phi)\gamma_+(f,\theta,\phi)]\widetilde{\alpha}(f)\\ & +[\gamma_+(f,\theta,\phi)\alpha_\times(f,\theta,\phi)-\gamma_\times(f,\theta,\phi)\alpha_+(f,\theta,\phi)]\widetilde{\beta}(f)\\ & +[\alpha_+(f,\theta,\phi)\beta_\times(f,\theta,\phi)-\alpha_\times(f,\theta,\phi)\beta_+(f,\theta,\phi)]\widetilde{\gamma}(f),\end{aligned} \tag{14}$$

has null response to the gravitational wave signal observed in (α, β, γ) when $(\theta, \phi) \to (\theta_s, \phi_s)$, regardless of the particular waveform considered.

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