

DETERMINATION OF METEOR SHOWERS ON OTHER PLANETS USING COMET EPHEMERIDES

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ABSTRACT

Meteor showers on Earth occur at well-known times and are associated with the decay of comets or other minor bodies whose orbital paths pass close to Earth's trajectory. On the surface, determining the closest proximity of two orbital paths appears to be a computationally intensive procedure. This paper describes a simple geometric method for determining the proximity between the orbital paths of two bodies (i.e., a comet and a planet) in the solar system from the known ephemerides of the objects. The method is used to determine whether meteor showers on other planets in the solar system could be associated with any of 250 known comets.

Key words: comets: general — meteors, meteoroids — solar system: general

1. INTRODUCTION

As they traverse their orbits about the Sun, comets slowly evaporate and fragment, leaving small bits of cometary debris along their orbital tracks. Some comet orbits intersect Earth's path, and the planet sweeps up a portion of these particulates each year. Generally, these particles are drawn into the atmosphere, where they burn up at high altitudes, producing the yearly meteor showers. A sample of the meteor showers expected on a regular basis for earth-bound observers is given in Table 1. A very detailed list of meteor streams encountered by Earth has been composed based on ground-based observations of amateur astronomers around the world (Jenniskens 1994).

Given the large number of meteor showers seen on Earth, it seems natural to ask about the possibility of meteor showers on other planets. It may be impractical for a sky observer of the future to view meteor showers from some worlds: Mercury has no atmosphere, the clouds of Venus are so thick that most meteors will likely burn up before a planet-bound observer could see them, Jupiter has no solid surface to sit on while viewing the shower, and so forth. Nevertheless, predicting regular meteor showers on other worlds may be important for protecting explorers and spacecraft from incoming particles, and it could be useful for planning expeditions and experiments to collect cometary material. It has been suggested (Adolfsson, Gustafson, & Murray 1996) that future Mars landers may be able to detect meteors from the surface of Mars.

A great deal of modern research has been devoted to analysis of the evolution of meteor streams in the solar system, particularly those that intersect Earth's orbit (for example, detailed analyses of the evolution of the Quadrantid stream can be found in Hughes, Williams, & Murray 1979, Williams, Murray, & Hughes 1979, and Murray, Hughes, & Williams 1980; the Geminid stream is analyzed by Fox, Williams, & Hughes 1982 and Jones 1985). These analyses take into account perturbations to the orbits of the parent bodies, as well as the subsequent evolution of the debris trail after the comet or minor body has continued on in its orbit. Over time, streams may wander into a planet's path, causing new meteor storms, or may wander out the planet's path, quenching a shower that has been periodic for decades or centuries (e.g., Murray et al. 1980 estimate that the Quadrantid shower will vanish by the year 2100).

To a first approximation, however, meteor showers will occur if the orbit of a planet and the orbit of a minor body intersect (or pass close to one another). One way to determine whether this occurs is to evolve the two orbits on a computer and watch for an intersection. A generalized method for finding the minimum separation between two orbits has been described by Murray et al. (1980), but the method reduces to a coupled set of equations for the orbital anomalies that requires numerical solution. Alternatively, the methods described in this paper approach the problem of determining the intersection of orbital planes in a completely analytical fashion, requiring only geometric methods and matrix algebra.

Section 2 describes the basic parameters and coordinate systems used to characterize orbits in this paper. Section 3 describes the rotations used to correctly orient two orbits with respect to each other and applies the rotations to essential vectors needed for the analysis. Section 4 uses the rotated vectors to determine the intersection between two orbital planes and computes the distance between the orbital paths when the planes intersect. Section 5 proposes a criterion for the existence of a meteor shower based on the distance between the orbits at intersection. The "time" of showers meeting the criterion is determined. Section 6 applies the condition of § 5 to 250 known comets, summarizes the results, and discusses the limitations of determining meteor showers using this method.

Throughout this paper, SI (*Système internationale*) units are employed, except where the size of the units makes it convenient to work in standard units employed in astronomy (e.g., on large scales astronomical units [AU] will be used, rather than meters).

2. DESCRIBING ORBITS

As is well known, one of the great discoveries of Johannes Kepler was that the planets travel on elliptical paths, with the Sun at one focus of the ellipse (Kepler's first law of planetary motion, published in 1609). Since then, an enormous body of knowledge has been developed regarding the analysis of orbital motion (see, e.g., Marion & Thornton 1988), allowing the determination of the position of virtually any object in the solar system at any moment in time.

For the work presented here, a time-dependent analysis

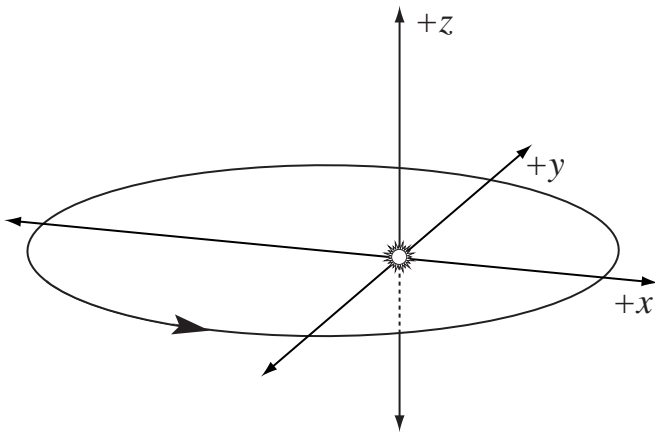


FIG. 1.—Reference coordinate system in the ecliptic plane. The z-axis is defined in the right-handed sense with respect to Earth's motion, and the x-axis points toward the perihelion of Earth.

of the orbital motion is not necessary.¹ The only information that is required is a knowledge of the trajectory of the orbit through space. The distance of the orbital path from the Sun may be written for elliptical orbits as

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity, and θ is the angle (the anomaly) between the body and the axis defined by perihelion, as measured in the orbital plane. The perihelion distance for the object can be found from equation (1) by taking $\theta = 0$, yielding

$$r_p = a(1 - e). \tag{2}$$

The distance expressed in equation (1) describes the correct size and shape of an elliptical orbit for any object around the Sun, but more information is needed to correctly orient the orbit in three-dimensional space. This information is typically collected in the orbital ephemeris, which expresses orbital parameters with respect to the plane that is coincident with the orbital plane of Earth (the ecliptic). This paper will use a (Cartesian) reference coordinate system defined in the ecliptic plane, as shown in Figure 1. The $+z$ axis is defined perpendicular to the ecliptic and in

¹ We are interested in knowing the proximity of two orbits when they cross. An interesting (but ultimately more difficult) question to address is whether two bodies might actually *collide* because their orbits intersect.

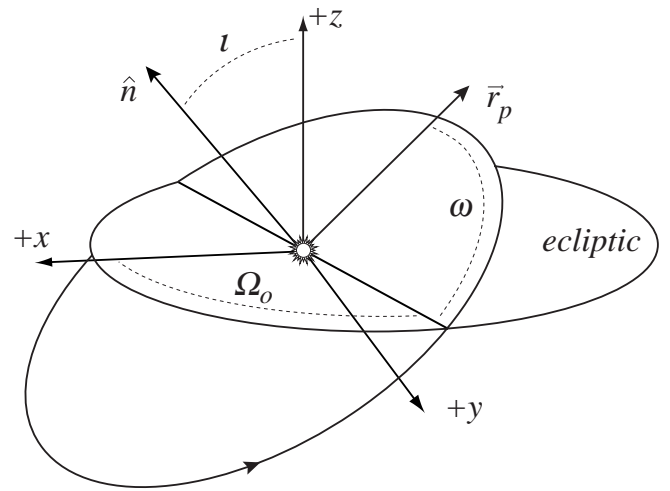


FIG. 2.—The three essential angles for correctly orienting orbits in three-dimensional space: (1) Ω_o , the (modified) longitude of the ascending node; (2) i , the inclination; and (3) ω , the argument of perihelion.

the right-handed sense with respect to Earth's orbital motion (i.e., when viewed looking down the $+z$ axis, Earth's motion is counterclockwise in the x - y plane). The $+x$ axis is defined along the direction of Earth's perihelion.

The orbital ephemeris of any body describes its orbit relative to the ecliptic plane and locates the object along its orbital path as a function of time. For the problem of determining the possible intersection of two orbital paths, only three elements of the full ephemeris for a body will be needed: Ω_o (a *modified* longitude of the ascending node), i (inclination), and ω (argument of perihelion). Each of these parameters is described below and shown in Figure 2.

The longitude of the ascending node, Ω , is the angle in the ecliptic plane between the vernal equinox (the first point of Aries) and the point at which the orbit crosses the ecliptic toward the $+z$ direction ("northward" across the ecliptic). The parameter, Ω_o , used in this paper is an offset longitude measured from the perihelion of Earth, rather than the first point of Aries (see Fig. 3). The inclination, i , is the angle between the normal vector of the orbit and the normal vector of the ecliptic. Last, the argument of perihelion, ω , is the angle between the position of the body as it crosses the ascending node and the position at perihelion, as measured in the *orbital plane of the body*.

In addition to these three angles, it will be useful to define two vectors for each orbit of interest: \hat{n} , the unit normal vector to the plane of the orbit, and r_p , the vector pointing to perihelion in the plane of the orbit.

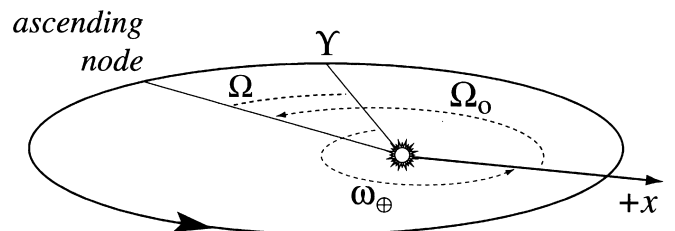


FIG. 3.—Modified longitude of the ascending node, Ω_o , defined in terms of Earth's argument of perihelion, ω_{\oplus} , and the conventional value of Ω for the orbit; Υ indicates the first point of Aries.

TABLE 1

SOME YEARLY METEOR SHOWERS SEEN FROM EARTH

Shower	Date
Quadrantids	Early January
Lyrids	Mid-April
η Aquarids	Early May
δ Aquarids	Late July
Perseids	Mid-August
Orionids	Mid-October
Leonids	Mid-November
Geminids	Mid-December

3. ROTATIONS FOR ORBITAL ORIENTATION

In order to correctly orient an orbit with respect to the ecliptic, assume (initially) that the orbit of interest is in the plane of the ecliptic, with the perihelion of the orbit aligned along the $+x$ axis (i.e., the orbit is co-aligned with Earth's orbit). A series of three rotations, based on the angles $\{\omega, i, \Omega\}$ from the orbital ephemeris, will produce the correct orientation. The first rotation will set the value of the ascending node with respect to perihelion, the second rotation will set the inclination to the ecliptic, and the third rotation will move the ascending node to the correct location in the ecliptic plane.

A useful method for describing rotations is in terms of matrices. While it is possible to construct a rotation matrix for rotations about a general axis, it is more convenient to conduct rotations about the coordinate axes shown in Figure 1. The matrices describing rotations about the x -, y -, and z -axes will be denoted $M_x(\phi)$, $M_y(\xi)$, and $M_z(\psi)$, respectively.

To demonstrate the rotations needed to orient the orbit, consider a general vector, \mathbf{A} , that is rigidly attached to the orbital plane, maintaining its orientation as the plane is rotated. The first rotation locates the ascending node with respect to perihelion; the rotation depends on the value of the argument of perihelion, ω . This is done by rotating around the z -axis by $\psi = \omega$. In terms of rotating a general vector \mathbf{A} , this can be written

$$\mathbf{A}_1 = M_z(\omega)\mathbf{A} . \quad (3)$$

When this operation is applied to the orbit, the ascending node will be located on the $+x$ axis.

The orbit is inclined around an axis that passes through the ascending node and through the Sun (at one focus of the orbit). Since the first rotation placed the ascending node on the $+x$ axis and the Sun lies at the origin of coordinates, a rotation around the x -axis by the inclination angle, $\phi = i$, will correctly incline the orbit. In terms of the vector \mathbf{A}_1 (resulting from eq. [3]), this yields

$$\mathbf{A}_2 = M_x(i)\mathbf{A}_1 . \quad (4)$$

Before the final rotation, it will be convenient to offset the longitude of the ascending node such that it is measured from the perihelion of Earth, rather than the vernal equinox (this makes the x -axis the origin for measuring the longitude of the ascending node). The angle between the vernal equinox and perihelion of Earth is simply the argument of perihelion for Earth, ω_\oplus , giving

$$\Omega_o = 2\pi - \omega_\oplus + \Omega \quad (5)$$

(see Fig. 3).

After the second rotation, the ascending node is still located on the $+x$ axis. Rotation about the z -axis by the offset longitude, $\psi = \Omega_o$, will rotate the longitude of the ascending node to its correct location in the ecliptic plane. In terms of the vector \mathbf{A}_2 (resulting from eq. [4]), this yields

$$\mathbf{A}_3 = M_z(\Omega_o)\mathbf{A}_2 . \quad (6)$$

The vector \mathbf{A}_3 (which is rigidly attached to the orbit) is correctly oriented with respect to the ecliptic.

The two vectors that will be of use later are the unit normal vector to the orbit, \hat{n} , and the perihelion vector, r_p . When the orbital plane is co-aligned with Earth's (before

any rotations have been performed), these vectors have the form

$$\hat{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad r_p = \begin{pmatrix} r_p \\ 0 \\ 0 \end{pmatrix} . \quad (7)$$

The rotation operations described by equations (3), (4), and (6) must be applied to these vectors so that they correctly describe the orbit with respect to the ecliptic. Conducting the rotation procedure yields

$$\hat{n}' = \begin{pmatrix} \sin i \sin \Omega_o \\ -\sin i \cos \Omega_o \\ \cos i \end{pmatrix}, \quad (8)$$

$$r_p' = r_p \begin{pmatrix} \cos \omega \cos \Omega_o - \sin \omega \cos i \sin \Omega_o \\ \cos \omega \sin \Omega_o + \sin \omega \cos i \cos \Omega_o \\ \sin \omega \sin i \end{pmatrix} . \quad (9)$$

4. INTERSECTION OF ORBITS

The procedure described in § 3 will correctly orient any orbit with respect to the ecliptic. One could take any planet's ephemeris (e.g., from the ephemerides given in Table 2) and construct the normal vector \hat{n} and perihelion vector r_p in accordance with equations (8) and (9).² Similar vectors could be generated for cometary ephemerides.

The real question of interest is not how the orbital planes of planets and comets are related to the ecliptic, but rather how they are oriented with respect to each other, and in particular where they intersect. The line defining the intersection of the orbital planes can be used to determine whether or not the orbits actually intersect.

Hereafter, assume that vectors related to a comet's orbit will bear the subscript "c" and vectors related to a planet's orbit will bear the subscript "+". Further, suppose the components of the normal vector for a comet's orbit are $\hat{n}_c = (a, b, c)$ and the components of the normal vector of a planet's orbit are $\hat{n}_+ = (e, f, g)$. Both orbital planes automatically share one point in common: the origin, which lies at the focus of each orbital ellipse. Given this point and the two vectors \hat{n}_c and \hat{n}_+ , the equations describing the two orbital planes are

$$\begin{aligned} \text{Comet plane: } & ax + by + cz = 0 ; \\ \text{Planet plane: } & ex + fy + gz = 0 . \end{aligned} \quad (10)$$

The intersection of the two planes is a line that is the common solution of the two expressions in equation (10). Using determinants, the common solution to these equations is found to be

$$x \begin{vmatrix} b & c \\ f & g \end{vmatrix}^{-1} = -y \begin{vmatrix} a & c \\ e & g \end{vmatrix}^{-1} = z \begin{vmatrix} a & b \\ e & f \end{vmatrix}^{-1} = k , \quad (11)$$

where k is an arbitrary constant. The solutions $\{x, y, z\}$ of equation (11) will be points along the line of intersection. It is convenient to use these values to define a new vector, λ ,

² To ease the notation, we will drop the primes for rotated vectors from here on. It will be understood that the normal vectors and perihelion vectors have been correctly oriented with respect to the ecliptic.

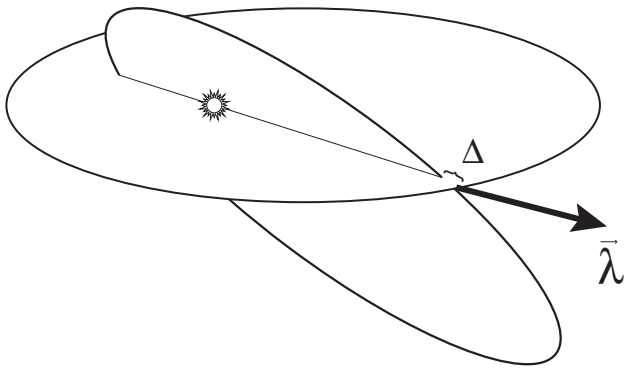


FIG. 4.—Intersection of two orbits, showing the essential quantities for defining the occurrence of a meteor shower: the separation of the orbits at crossing, Δ , and the node vector, λ , which defines the intersection of the two orbits and is used to determine the time of the meteor shower.

called the “node vector” (see Fig. 4). It points along the line of nodes (the intersection of the two planes) and has components

$$\lambda = k \begin{pmatrix} bg - cf \\ ce - ag \\ af - be \end{pmatrix}. \tag{12}$$

To determine whether the orbital paths intersect, one must know the radii of the orbits along the line of nodes. An orbital radius may be determined from equation (1) if the value of the anomaly, θ , is known. In terms of two orbits inclined with respect to each other, the angles of interest will be the angle between the perihelion vector for each orbit, r_p , and the node vector, λ . For each orbit, the angle is defined in terms of the dot product of the two vectors, yielding

$$\cos \theta = \frac{r_p \cdot \lambda}{|r_p| |\lambda|}. \tag{13}$$

The orbits have two opportunities to intersect: at the ascending node, and at the descending node. Equation (13) gives the angle at a single node. To obtain the value of the anomaly at the other node, dot the perihelion vector, r_p , into the negative of the node vector, $-\lambda$.

Once the anomaly is known, the distance between the orbital paths when the planes intersect is simply

$$\Delta = |r_+ - r_c|, \tag{14}$$

where r_+ and r_c are computed using equation (1), with the anomaly defined by equation (13) and the appropriate orbital parameters derived from tabulated ephemerides.

5. IS THERE A METEOR SHOWER?

The occurrence of a meteor shower associated with a particular comet will depend on the value of the separation between the orbital paths, Δ . A variety of proximity criteria could be developed, depending on specific considerations one would like to make regarding the likelihood that particles from a given stream might reach a planetary body (such as the proximity of the mean orbits of a comet and planet, or the spread of cometary particles around the orbit of the parent comet). The basic criterion may be expressed such that

$$\Delta \leq \hat{r}, \tag{15}$$

where \hat{r} is the minimum separation one considers likely for a shower to occur.

In this paper, the criterion for an orbital intersection’s causing a meteor shower will be scaled to a region where the gravitational potential of the planet dominates over the gravitational potential of the Sun. The criterion will be

$$\Delta \leq \kappa R_l, \tag{16}$$

where R_l is the “Roche lobe radius” (defined as the radius of a sphere that has the same volume as the planet’s Roche lobe) and κ is an arbitrary adjustable scaling factor. The Roche lobe radius can be approximated by

$$R_l \sim 0.52a \left(\frac{m_+}{M_\odot + m_+} \right)^{0.44}, \tag{17}$$

where m_+ and M_\odot are the mass of the planet and the Sun, respectively, and a is the semimajor axis of the planet’s orbit (Iben & Tutukov 1984).

Once an intersection (in the sense of eq. [16]) has been found, one would like to know *when* the associated meteor shower might occur, so that it is possible to mount an observational effort to detect the shower. A good marker for a planetary encounter with the meteor stream is the value of the planetary anomaly along the line of nodes, given by equation (13).

Given a particular value of the anomaly, θ , it is possible to write a closed-form expression $t(\theta)$ for the time at which the planet will arrive at that point in its orbit. The areal velocity may be written

$$\frac{dA}{dt} = \frac{\pi ab}{\tau}, \tag{18}$$

where a and b are the semimajor and semiminor axes, respectively, and τ is the period of the orbit. Separating equation (18) and writing the area element as $dA = \frac{1}{2}r^2 d\theta$

TABLE 2
MEAN EPHEMERIDES FOR THE PLANETS OF THE SOLAR SYSTEM (EPOCH J2000)

Planet	a (AU)	e	i (deg)	Ω (deg)	ω (deg)
Mercury	0.38709893	0.20563069	7.00487	48.33167	29.12478
Venus	0.72333199	0.00677323	3.39471	76.68069	54.85229
Earth	1.00000011	0.01671022	0.00005	-11.26064	114.20783
Mars	1.52366231	0.09341233	1.85061	49.57854	286.4623
Jupiter	5.20336301	0.04839266	1.3053	100.55615	-85.8023
Saturn	9.53707032	0.0541506	2.48446	113.71504	-21.2831
Uranus	19.19126393	0.04716771	0.76986	74.22988	96.73436
Neptune	30.06896348	0.00858587	1.76917	131.72169	-86.75034
Pluto	39.348168677	0.24880766	17.14175	110.30347	113.76329

TABLE 3
RESULTS OF A METEOR-SHOWER SEARCH USING THE COMETS IN JPL'S DASTCOM

Comet	Δ/R_i	θ (deg)	$r(\theta)$ (days)
Intersections with $\Delta \leq R_i$			
Earth:			
109P/Swift-Tuttle	0.5043	216.49717	220.81696
Jupiter:			
P/LONEOS-Tucker (1998 QP ₅₄)	0.02759	143.72224	1688.24555
117P/Helin-Roman-Alu 1	0.05761	54.79245	605.7507
43P/Wolf-Harrington	0.10689	241.59541	2965.94337
P/Hergenrother (1998 W2)	0.13008	158.6528	1883.37507
C/Hale-Bopp (1995 O1)	0.15907	267.71691	3287.20798
78P/Gehrels 2	0.2102	206.19238	2510.80676
75P/Kohoutek	0.21718	256.88227	3155.62992
P/Spahr (1998 W1)	0.24054	267.30471	3282.24601
124P/Mrkos	0.32574	344.80757	4164.72657
14P/Wolf	0.32874	191.77241	2321.02269
53P/Van Biesbroeck	0.37063	143.86166	1690.05544
59P/Kearns-Kwee	0.39771	294.48157	3602.23635
91P/Russell 3	0.41994	56.45964	624.68681
76P/West-Kohoutek-Ikemura	0.44039	248.71583	3054.87529
26P/Grigg-Skjellerup	0.45251	21.59907	236.07478
132P/Helin-Roman-Alu 2	0.50932	176.74286	2122.18424
P/Kushida (1994 A1)	0.54051	239.05142	2933.93263
16P/Brooks 2	0.57961	175.77613	2109.39006
83P/Russell 1	0.59462	34.84717	382.18644
D/Kowal-Mrkos (1984 H1)	0.61747	57.35711	634.89823
135P/Shoemaker-Levy 8	0.68357	28.92759	316.71554
139P/Vaisälä-Oterma	0.72789	241.18336	2960.76693
86P/Wild 3	0.73742	55.46464	613.3804
104P/Kowal 2	0.81618	233.8782	2868.46755
P/LINEAR-Mueller (1998 S1)	0.87419	157.76011	1871.64355
P/Shoemaker-Levy 6 (1991 V1)	0.87826	199.12624	2418.00496
18P/Perrine-Mrkos	0.94922	228.33725	2797.82077
85P/Boethin	0.97138	164.48654	1960.19405
Saturn:			
P/Jäger (1998 U3)	0.06673	210.03052	6366.0197
126P/IRAS	0.53161	83.17079	2299.91062
Uranus:			
C/Li (1999 E1)	0.84435	137.45475	11360.58497
Intersections with $\Delta \leq R_i$			
Earth:			
55P/Tempel-Tuttle	4.15373	312.31161	318.28395
26P/Grigg-Skjellerup	4.518	110.36191	110.13972
Mars:			
C/LINEAR (1998 U5)	1.98779	90.14579	151.61158
Jupiter:			
47P/Ashbrook-Jackson	1.00145	162.12665	1929.08903
15P/Finlay	1.00753	186.90971	2256.74179
97P/Metcalf-Brewington	1.09545	175.71325	2108.55792
81P/Wild 2	1.10168	320.59158	3897.71867
121P/Shoemaker-Holt 2	1.10447	264.90153	3253.24816
54P/de Vico-Swift	1.1133	152.93248	1808.3284
56P/Slaughter-Burnham	1.1211	143.93726	1691.03684
P/Korlevic-Juric (1999 DN ₃)	1.15484	347.38897	4192.9561
P/Mueller 4 (1992 G3)	1.16212	312.27537	3804.66421
52P/Harrington-Abell	1.20057	316.89669	3856.48162
69P/Taylor	1.2392	274.64237	3370.06049
P/Larsen (1997 V1)	1.269	224.13556	2743.90598
46P/Wirtanen	1.31801	245.23688	3011.54806
100P/Hartley 1	1.32008	21.70502	237.23779
87P/Bus	1.34382	15.87435	173.3314
C/Ferris (1999 K2)	1.3836	285.50894	3498.14812
77P/Longmore	1.43731	358.02608	4309.06758

TABLE 3—Continued

Comet	Δ/R_i	θ (deg)	$t(\theta)$ (days)
C/Mueller (1997 J1)	1.44651	262.26797	3221.3349
P/Hartley-IRAS (1983 V1)	1.51103	166.77415	1990.37975
119P/Parker-Hartley	1.53129	236.51823	2901.93741
114P/Wiseman-Skiff	1.54424	256.92325	3156.13216
102P/Shoemaker 1	1.58558	143.00259	1678.90784
P/Spahr (1998 U4)	1.61195	169.07697	2020.79437
33P/Daniel	1.64668	229.96736	2818.65939
62P/Tsuchinshan 1	1.6969	261.53064	3212.37462
70P/Kojima	1.78536	288.99826	3538.80021
60P/Tsuchinshan 2	1.84257	272.21092	3341.08206
4P/Faye	1.91557	192.45922	2330.09358
67P/Churyumov-Gerasimenko	1.95263	207.28864	2525.16175
6P/d'Arrest	2.01111	126.65145	1468.85806
36P/Whipple	2.02544	175.2223	2102.06112
C/LINEAR (1998 U1)	2.08822	12.83484	140.08603
C/Spacewatch (1997 BA ₆)	2.15772	302.67334	3696.04414
P/Levy (1991 L3)	2.21811	131.97724	1536.80051
116P/Wild 4	2.27558	347.04647	4189.21226
P/Shoemaker-Levy 1 (1990 V1)	2.39831	215.04783	2626.37534
9P/Tempel 1	2.41105	50.10947	552.78419
C/LINEAR (1999 H3)	2.59549	138.47987	1620.39097
31P/Schwassmann-Wachmann 2	2.63332	303.86438	3709.59249
P/LINEAR (1999 J5)	2.69947	98.43266	1117.77483
C/LINEAR (1998 W3)	2.71291	108.75604	1244.38557
21P/Giacobini-Zinner	2.73222	182.72852	2201.41502
40P/Väisälä 1	2.84407	304.29294	3714.46212
108P/Ciffréo	2.88171	214.58058	2620.30119
136P/Mueller 3	2.92721	128.57415	1493.32993
42P/Neujmin 3	2.93144	153.24921	1812.47528
7P/Pons-Winnecke	3.13747	78.25359	876.539
103P/Hartley 2	3.19169	209.67222	2556.32919
128P/Shoemaker-Holt 1-B	3.22265	213.44774	2605.56299
D/van Houten (1960 S1)	3.25993	336.58556	4074.58242
P/Kushida-Muramatsu (1993 X1)	3.26773	250.65642	3078.93931
C/Zhu-Balam (1997 L1)	3.26781	218.82993	2675.43314
P/LINEAR (1998 VS ₂₄)	3.26871	158.78542	1885.11843
P/Spahr (1998 U4)	3.29681	349.07697	4211.40146
61P/Shajn-Schaldach	3.3833	164.21702	1956.63978
65P/Gunn	3.43747	49.5381	546.34347
120P/Mueller 1	3.59615	161.60426	1922.20873
129P/Shoemaker-Levy 3	3.6607	284.2452	3483.36888
30P/Reinmuth 1	3.66617	288.52425	3533.29093
22P/Kopff	3.67008	113.53434	1303.71706
112P/Urata-Nijima	3.69814	194.43253	2356.14237
110P/Hartley 3	3.70194	272.28798	3342.00224
49P/Arend-Rigaux	3.78256	288.50492	3533.06618
P/Lagerkvist (1996 R2)	3.9786	175.43706	2104.90302
19P/Borrelly	4.00335	239.67933	2941.845
17P/Holmes	4.00357	130.61884	1519.42495
137P/Shoemaker-Levy 2	4.01952	229.51888	2812.93064
131P/Mueller 2	4.05027	208.10376	2535.82718
P/Jedicke (1995 A1)	4.07511	102.29178	1164.85082
D/Tritton (1978 C2)	4.234	282.865	3467.19321
48P/Johnson	4.31823	104.30891	1189.57815
106P/Schuster	4.41102	212.99144	2599.62197
C/LINEAR (1998 M5)	4.67076	318.4884	3874.26616
P/LONEOS (1999 RO ₂₈)	4.84285	141.12541	1654.58458
98P/Takamizawa	4.88896	113.76163	1306.55052
C/Spacewatch (1997 P2)	4.89278	286.39551	3508.49848
73P/Schwassmann-Wachmann 3	4.9158	51.54895	569.03117
P/Helin-Lawrence (1993 K2)	4.99431	76.00361	850.14018
Saturn:			
C/Catalina (1999 F1)	1.01329	107.67064	3035.62842
P/Gehrels (1997 C1)	1.27361	164.60138	4862.59895
P/Hermann (1999 D1)	1.35745	252.08077	7703.6729

TABLE 3—Continued

Comet	Δ/R_i	θ (deg)	$t(\theta)$ (days)
C/LINEAR (1998 Q1).....	1.36923	70.34971	1928.09742
P/Shoemaker 4 (1994 J3)	2.45512	358.40089	10704.16966
P/Montani (1997 G1)	2.89611	185.28975	5549.17747
P/Helin (1987 Q3)	3.16833	76.81205	2114.42428
63P/Wild 1	3.2706	260.23561	7952.44828
140P/Bowell-Skiff.....	3.30576	231.84877	7070.65503
D/Bradfield 1 (1984 A1).....	3.57748	262.73062	8027.80737
134P/Kowal-Vavrova	3.64885	319.9063	9665.71078
C/Spacewatch (1997 BA ₆).....	3.95068	44.94144	1214.46702
C/LINEAR (1999 N4).....	3.95944	78.38428	2160.08505
C/LINEAR (1999 H3).....	4.19679	241.0887	7362.46033
P/Lagerkvist-Carsenty (1997 T3).....	4.90066	120.32592	3428.86878

allows one to write an integral for the time t as

$$t(\theta) = \left(\frac{\tau}{\pi ab}\right) \frac{1}{2} \int_0^\theta r^2 d\theta'. \tag{19}$$

Using the shape equation (eq. [1]) to write $r = r(\theta)$, and using $b = a(1 - e^2)^{1/2}$ to express the semiminor axis, equation (19) can be integrated to give

$$t(\theta) = \frac{\tau}{2\pi} \left[2 \tan^{-1} \left(\sqrt{\frac{1-e}{1+e}} \tan \frac{\theta}{2} \right) - \frac{e\sqrt{1-e^2} \sin \theta}{1+e \cos \theta} \right], \tag{20}$$

which is the time it takes the planet to pass from perihelion ($\theta = 0$) to the anomaly value θ .

6. RESULTS AND DISCUSSION

As an example of the methods presented here, a search for comet-planet orbital intersections in the solar system was carried out using the planetary ephemerides shown in Table 2 (Standish et al. 1992) and the cometary ephemerides provided in the Jet Propulsion Laboratory's DASTCOM (Database of Asteroids and Comets).³ DASTCOM is a collection of orbital parameters and physical characteristics for the numbered asteroids, unnumbered asteroids, and periodic comets, used for analyses of solar system dynamics. DASTCOM includes all known comets with periods less than 200 years (periodic comets), and long-period comets that have made a return after 1995 (there are currently 73 long-period comets in DASTCOM, and 208 comets with periods less than 200 yr). The orbital elements given in DASTCOM are osculating elements, computed from observations during the comet's most recent apparition.

The results of the search for comet-planet orbital intersections are listed in Table 3, which specified an encounter distance of

$$\Delta \leq 5R_i. \tag{21}$$

In all, 128 possible showers were detected: three at Earth, one at Mars, 106 at Jupiter,⁴ 17 at Saturn, and one at Uranus. If one reduces the encounter distance to $\Delta \leq R_i$, only 32 possible showers are detected (shown in the upper portion of Table 3): one at Earth, 28 at Jupiter, two at

Saturn, and one at Uranus. If one allows the encounter distance to expand to $\Delta \leq 10R_i$, 188 possible showers are detected (not shown in Table 3): four at Earth, five at Mars, 148 at Jupiter, 24 at Saturn, six at Uranus, and one at Neptune.

Comets with orbital scales smaller than the solar system ("short-period comets") have evolved largely under the influence of perturbations due to Jupiter (the mass of Jupiter is greater than the mass of the other planets combined), yielding a large population of comets that cross Jupiter's orbit. The search for the origin of these "Jovian family comets" has been a matter of much numerical simulation and debate (see, e.g., Quinn, Tremaine, & Duncan 1990). The disproportionately large number of showers detected for Jupiter can be attributed to this feature of the comet population.

A good check of the procedure described in this paper is to consider the predicted showers at Earth. In particular, the method outlined in this work predicts two meteor streams that can be identified with known showers. The first is the stream from comet Tempel-Tuttle, occurring at $t \sim 318$ days. This stream can be identified with the Leonid meteor shower (known to be a stream from Tempel-Tuttle), which occurs in mid-November each year. The second is a stream from comet Swift-Tuttle, occurring at $t \sim 221$ days. This stream can be identified with the Perseid meteor shower (known to be a stream from Swift-Tuttle), which occurs in mid-August each year.

The possible showers computed here have all assumed that the orbits of the comets are static and do not precess. Further, it is assumed that the meteor streams remain attached to those static orbits without wandering under the influence of gravitational perturbations in the solar system. In addition, the influences of "local" bodies around each planet (e.g., Earth's moon, or the Galilean satellites around Jupiter) have been ignored. Nevertheless, the method provides a useful way for determining the possibility that a given planet will encounter a meteor stream from minor bodies in the solar system.

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³ In 1999; see <http://ssd.jpl.nasa.gov/dastcom.html>.

⁴ In fact, the results of Table 3 show that Jupiter's orbit intersects the path of comet P/Spahr (1998 U4) *twice*: one intersection at a separation of $\Delta \approx 1.6R_i$, and a second intersection (at the other node) with a separation of $\Delta \approx 3.3R_i$.

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