Black Holes

What is a Black Holes

• Let us first agree on the definition of what we mean by black hole, where definition means " what is its defining physical characteristic that makes us say *this is a black hole*?"

• The fundamental definition is this: ***a black hole is an astrophysical object whose escape velocity is the speed of light***

• It is important to note that by this definition, *black holes have nothing to do with relativity.* Black holes have a lot to do with *gravity.*

• If we agree that gravity is an attractive force between masses (which Newtonian gravity certainly is), *then black holes are a natural consequence of all gravitational theories*

• The reason is that if gravity is always attractive, then the only thing that can prevent gravitational collapse is opposing forces from other physical effects. We have seen this:

- ▷ In *ordinary stars*, the thermal pressure from ongoing nuclear burning opposes gravity's attempt to collapse the star
- ▷ In *white dwarfs*, gravity collapsed the star until electron degeneracy pressure provided enough outward pressure to stop the collapse
- ▷ In *neutron stars*, gravity collapses the star even further until neutron degeneracy pressure provided enough outward pressure to stop the collapse

• You could imagine that if you continued to pile matter on top of a neutron star, that it should be possible to overcome the neutron degeneracy pressure. There is no known force beyond neutron degeneracy pressure that could oppose the gravitational collapse. Beyond neutron stars, gravitational collapse forms ***black holes***

• The first theoretical description of black holes was written down by the Reverend John Michell, who in 1784 wrote a letter to Henry Cavendish describing a *dark star*, and object which had been shrunk to the point where the surface escape velocity was the speed of light. This idea was rediscovered (with much more press) by Pierre Simon Laplace in 1796.

• We can understand the properties of such an object if we star with the definition of escape velocity. The radius would be:

$$v_{esc} = \sqrt{\frac{2GM}{r}} \qquad \rightarrow \qquad r = \frac{2GM}{v_{esc}^2} \qquad \rightarrow \qquad r = \frac{2GM}{c^2}$$

• Remember: *we derived the v_{esc} formula from Newtonian gravity.* This is an entirely Newtonian result!

• The savvy reader will notice that the size we have derived here is equivalent to the classical *Schwarzschild radius* from general relativity. This should not be surprising because we know that Newtonian gravity is a part of general relativity (the weak field limit).

• In general relativity, the “surface” of the black hole (located at the Schwarzschild radius for non-spinning black holes) is called the *event horizon.* It is the distance from the black hole where the gravity becomes so strong that $v_{esc} = c$

Looking for Black Holes: Cygnus X1

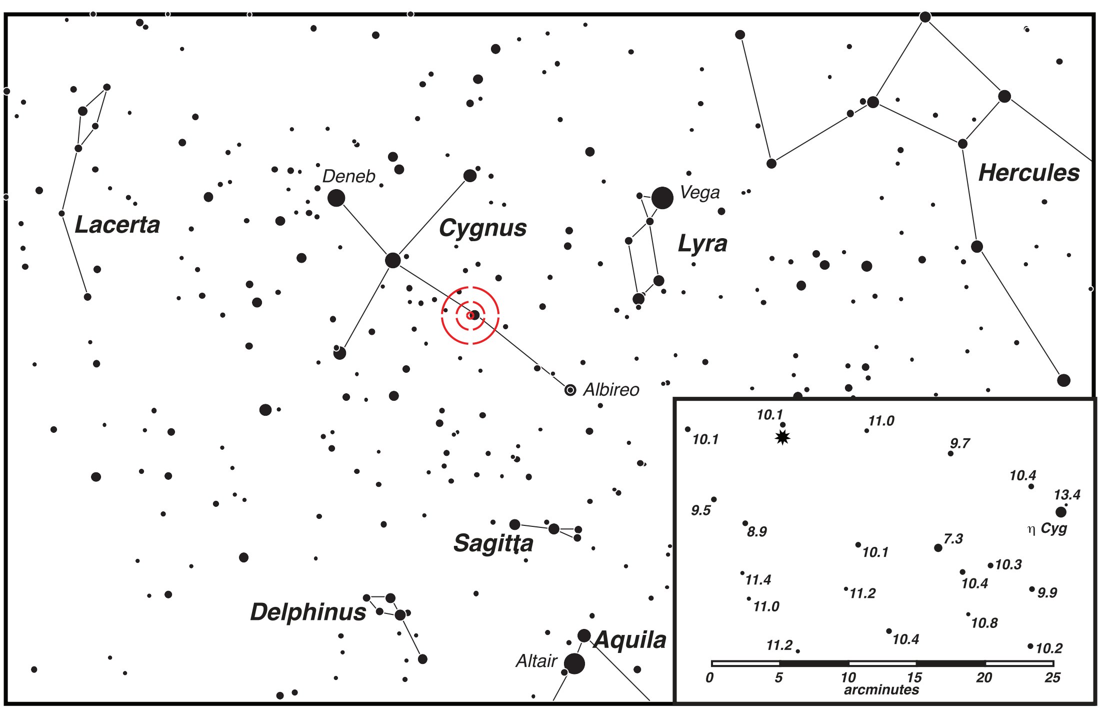
• It is fine to say black holes exist, but as observers we must ask the question: *can we find any?*

• There is a fundamental problem here, namely that by definition, black holes emit no light? So what do we do? *We look for the effect of black holes on things that do emit light!*

• The closest black hole candidate to the Earth is called *Cygnus X-1.* Located in the sky near the naked eye star η Cygni, there is an 8.8 magnitude blue supergiant (spectral type O9) visible in the telescope. It is approximately 6100 lightyears away

• The source is extremely bright in x-rays, a completely anomalous behaviour for an O9 supergiant.

• The star also shows an astrometric wobble, with a period of $P = 5.599829$ days. Analysis has suggested the periodicity is an orbital periodicity ($i \simeq 48°$, $a \simeq 0.2$ AU, $e \simeq 0.0018$).



• We can use our know-how to figure out the properties of the unseen companion. Start with Kepler III:

$$GM_{tot} = \frac{4\pi^2}{P^2}a^3 \qquad \rightarrow \qquad M_{tot} = \frac{4\pi^2}{GP^2}a^3$$

• Suppose we assume $M_{tot} = M_{O9} + M_\bullet$, and $M_{O9} \simeq 27M_\odot$ (BOB, Appendix G). Treating this as a circular orbit with $r \simeq a$ then yields

$$M_{tot} \simeq 34M_\odot \qquad \to \qquad M_\bullet \simeq 7M_\odot$$

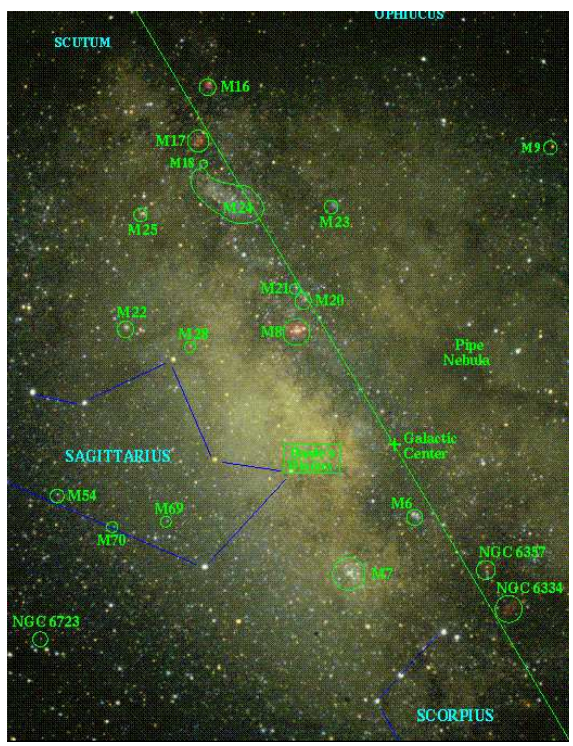
• Now think about the x-rays. One of the fundamental tenets of astrophysics is that hot gas emits energetic light. These two objects are close enough that they are gravitationally interacting. They are only 0.2 AU apart and the $7M_\odot$ object cannot be seen, so assume it is *compact.* Again appealing to BOB Appendix G, we estimate the size of the blue supergiant to be $R_{O9} \simeq 27R_\odot = 0.1255AU$.

• Suppose a small parcel of gas falls from the surface of the O9 supergiant to the location of the compact object. As a first order approximation, assume all of the potential energy the gas might have is converted into kinetic energy, which is then converted into thermal energy which if it is emitted as light is the energy of the emitted photon: $U_E = K_E = E_{th} = E_\gamma$. So, starting with potential energy

$$E_\gamma = U_E \qquad \to \qquad \frac{hc}{\lambda} = \frac{GM_\bullet m_h}{d_\bullet} \qquad \to \qquad \lambda = \frac{hcd_\bullet}{GM_\bullet m_h} = 1.43\ nm \text{ (xray)}$$

• Look at what we have done here. To get the x-rays, we assumed the gas had to fall *all the way to the location of the unseen object* suggesting it has to be extremely compact. Looking at the spectra of your x-rays then is a way to place loose bounds on the size of the object

Looking for Black Holes: Sgr A⋆

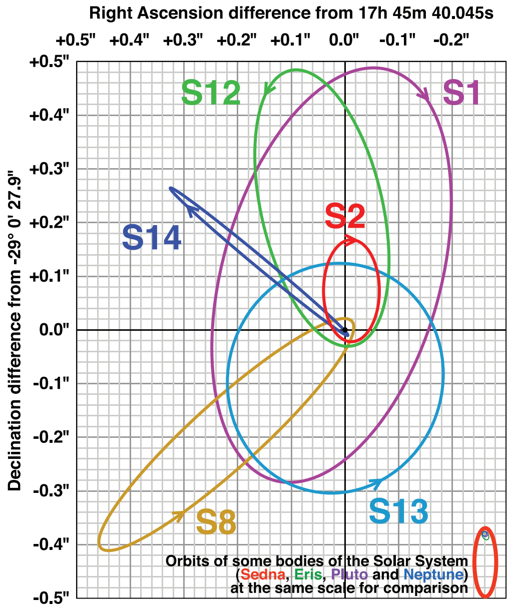


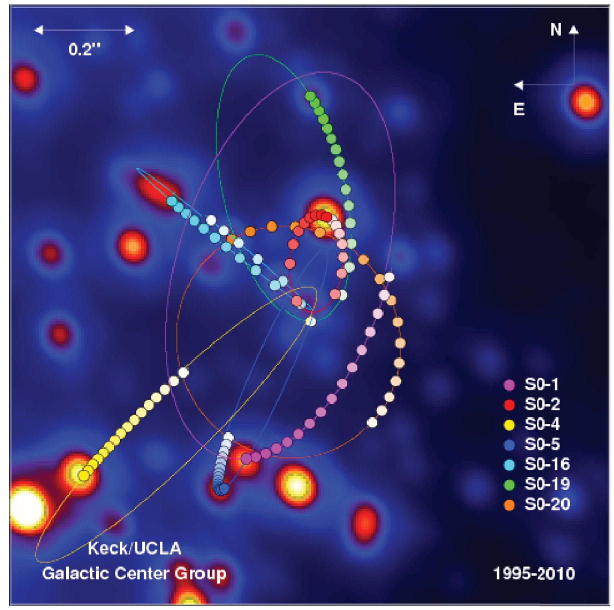
• Our view along the galactic plane is obscured in the optical by intervening gas and dust. Toward the galactic center there is an area where the obscuration and interstellar dust is lower, known as Baade's Window (named after the astronomer, Walter Baade).

• At infrared and radio wavelengths, the data show there is a complex of bright gas and dust in the vicinity of the center, several tens of lightyear across, called *SgrA* (Sagittarius A). It consists of three components, the supernova remnant Sgr A East, the spiral structure Sgr A West, and a very bright compact radio source at the center of the spiral, Sgr A⋆. These three overlap: Sgr A East is the largest, West appears off-center within East, and Sgr A⋆ is at the center of West.

• Detailed observations in the infrared have revealed a small cluster of stars in the vicinity of Sgr A⋆. We have been observing this cluster (generally called the *SCluster*) since the

1990s, and now have a long enough observational record that we can *map out the orbits of individual stars!*





• Of particular interest in this cluster is the star $S02$, which has as eccentricity of $e = 0.87$ and an orbital period of $P_{S02} = 15.2$ yr — we've seen it complete an entire orbit around an unseen, dark mass. In 2002, we saw $S02$ pas through periapsis, at a distance of $r_p = 120\ AU = 1.8 \times 10^{13}\ m$. During periapsis passage, it was traveling at $v \sim 5000$ km/s!

• We can use the orbit to derive the properties of the unseen mass it is orbiting. The definition of r_p:

$$r_p = a\,(1-e) \qquad \rightarrow \qquad a = \frac{r_p}{1-e} = 1.4 \times 10^{14}\ m$$

• Using this in Kepler III, we can find the mass of Sgr A$\star$:

$$G(M_{sgrA\star}) = \frac{4\pi^2}{P^2}a^3 \qquad \rightarrow \qquad M_{sgrA\star} \simeq \frac{4\pi^2}{GP^2}a^3 = 6.82 \times 10^{36}\ kg = 3.4 \times 10^6 M_\odot$$

• The current best estimate of the mass of Sgr A$\star$ from detailed analysis of the postion of $S02$ gives a mass estimate of $M_{sgrA\star} = 4.1 \pm 0.6 \times 10^6 M_\odot$ (Ghez et al., ApJ **689**, 1044 [2008]).

Black Holes in Other Galaxies

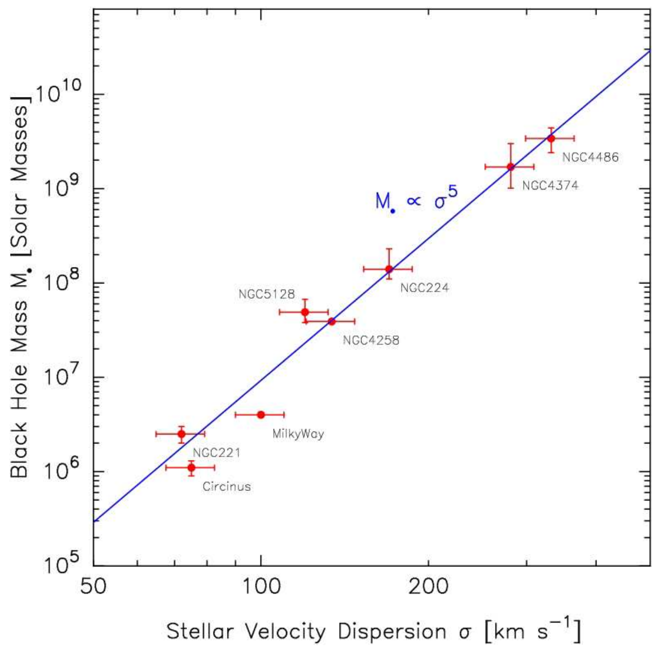
• Why does the Milky Way have a black hole at its center? Is it unique among galaxies, or are black holes ubiquitous?

• Careful observations of many galaxies has revealed that many galaxies harbor black holes in their cores! The question is whether or not there is an easy way to determine if a galaxy has a massive black hole at its center or not. As it turns out, there is.

• One of the easiest things to measure about a galaxy is the distribution of stellar speeds in galactic cores (for example, using doppler shifts).

• If you measure the speed of many stars, you can analyze the statistics of the speeds. They will have some average, and some *spread* around that average. The spread is related to a quantity called the *velocity dispersion*, σ.

• Careful observations have shown that the velocity dispersion σ is correlated with the black hole mass $M_\bullet$. The larger the black hole, the larger the velocity dispersion.



• As you can see from the graph, the relationship is a power law, $M_\bullet \propto \sigma^\alpha$. This is called the $M - \sigma$ relation. The current best fit is

$$\log\left(\frac{M_\bullet}{M_\odot}\right) = 8.12 + 4.24 \log\left(\frac{\sigma}{200\ km/s}\right)$$

• Why the $M - \sigma$ relation exists is still a matter of intense research, but the heuristic explanation has to do with interactions between stars and the black hole. Generically speaking, when the stars in the core of galaxy have high velocity dispersion, there is a bulge, leading us to conclude that *galaxies with bulges have black holes*.

• This is true, irrespective of galaxy type. Many bulgeless spirals (such as the Triangulum Galaxy, M33) appear to have no black hole, while many large ellipticals are "all bulge" and have very large black holes (such as M87, with a ~ 2 billion $M_\odot$ black hole)

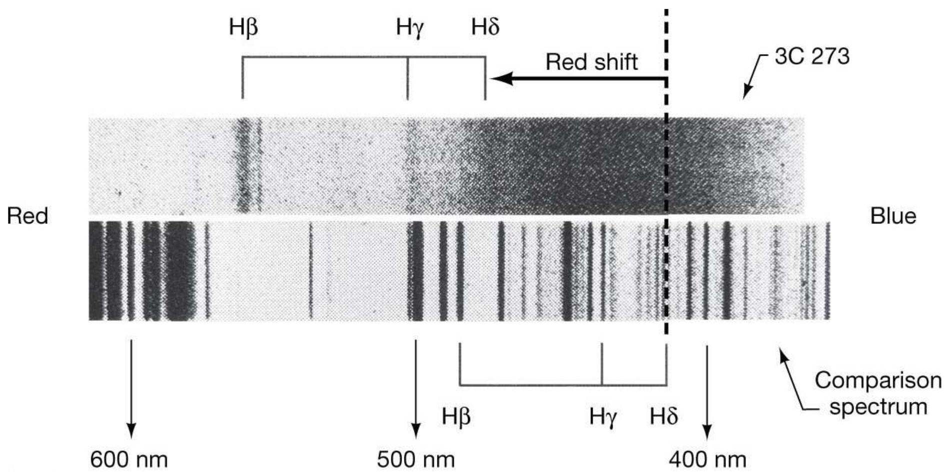
Quasars

• In the late 1950s radio astronomers discovered many bright, point like radio sources with no discernible optical counterparts on the sky. Many of these were later catalogued and included in the *Third Cambridge Catalogue*.

• ***3C48*** was identified with a counterpart in 1960, but the spectrum showed an odd array of *unknown broad emission lines*.

• ***3C273*** was occulted by the Moon five times in 1962, allowing precise identification of an optical counterpart. Again the spectrum showed an odd array of *unknown broad emission lines*.

• It was realized that the lines in 3C273 were actually hydrogen lines with *extreme redshifts*. The redshift was equivalent to the shift produced by a recessional velocity of 47,000 km/s, or 15.8%× the speed of light! Going back to 3C48, it was discovered to have a recessional velocity of 37%× the speed of light!



• As we will see when we study cosmology, the high redshift implies that these quasars are *enormously far away*. The estimated distance of 3C273 is 620 Mpc, and it has an apparent magnitude of $m = 12.8$. Using the distance modulus formula then

$$m - M = 5 \log\left(\frac{d}{10\ pc}\right)$$

Putting in the numerical values

$$M = m - 5\log\left(\frac{d}{10\ pc}\right) = 12.8 - 5\log\left(\frac{640 \times 10^6\ pc}{10\ pc}\right) = -26.23$$

• So 3C273 has an *absolute magnitude* roughly equivalent to the *apparent magnitude* of the Sun. If the quasar were 10 pc from Earth, it would look as bright as the Sun in the daytime sky! We can use the absolute magnitude to convert to *luminosity*, the energy per unit time output by the quasar:

$$M_1 - M_2 = -2.5\log\left(\frac{L_1}{L_2}\right) \qquad \rightarrow \qquad M_1 - M_\odot = -2.5\log\left(\frac{L_1}{L_\odot}\right)$$

Solving for L_1, the luminosity of the quasar

$$L_1 = 10^{(M_\odot - M_1)/2.5} L_\odot = 10^{(4.74+26.23)/2.5} L_\odot = 2.6 \times 10^{12} L_\odot$$

• The luminosity of 3C273 is almost ***3 trillion*** times the luminosity of the Sun! What could possibly be providing the energy for these objects?

Eddington Luminosity

• When we considered the x-ray luminosity of Cygnus X-1, we had considered the flow of gas onto a black hole, equating the loss of potential energy with the emitted photon energy as a parcel of gas fell toward the black hole.

• There is a limit to how well this mechanism will work, defined by the fact that as gas falls inward the gravitational force will be countered by an outward force provided by the radiative pressure of the emitted photons.

• For a luminosity L, the *flux* at a distance r away is

$$f = \frac{L}{4\pi r^2}$$

• Bear in mind the meaning of the luminosity here — L is the power, the rate at which energy is radiated away from the quasar.

• Recall that for photons, the *momentum* is related to the *energy* by $p = E/c$. A useful construct is to think about the radiative energy being carried away (the flux F) in terms of *how much momentum is being carried away.* This is called the *momentum flux*, $\mathcal{F}_p$

$$\mathcal{F}_p = \frac{1}{c} f = \frac{L/c}{4\pi r^2}$$

• Since L was the rate at which energy is radiated away from the quasar, then L/c is the rate at which momentum is radiated away from the quasar. You might call it the "momentum luminosity" and the momentum flux then is the "momentum luminosity per unit area." But what is *momentum luminosity*?

• You may recall that an important form of Newton's second law defines force not in terms of acceleration, but rather in terms of *the rate at which momentum changes*:

$$F = m \cdot a \qquad \rightarrow \qquad F = \frac{dp}{dt}$$

• *Momentum luminosity* is just a force! Dimensionally then, the momentum flux is a *pressure*, so multiplying it by a characteristic area is a force. The area of interest here is the *interaction cross-section* with the particles that the outward flux of photons will encounter. In this case, the particle targets are atoms, and the light strongly interacts with the electrons.

• Astronomers describe the interaction of light and electrons by *Thomson scattering*, which physicists will recognize as low-energy *Compton scattering*. Without deriving the result, the cross-section for electrons is given by

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_o m_e c^2}\right)^2 = 6.65\times 10^{-29}\ \mathrm{m}^2$$

• So the *radiative force* provided by a source with luminosity L is given by

$$F_{rad} = \sigma_e \mathcal{F}_p = \frac{\sigma_e}{c}\frac{L}{4\pi r^2}$$

• This outward force is countering the inward pull of gravity, which is acting on entire atoms of mass $m = m_p + m_e \simeq m_p$, so the gravitational force provided by the black hole is

$$F_{grav} = G\frac{M_\bullet m_p}{r^2}$$

• The *Eddington Luminosity*, L_{Edd}, is the luminosity required for the gravitational force to be balanced by the outward radiative pressure:

$$F_{rad} = F_{grav} \qquad \rightarrow \qquad L_{Edd} = \frac{4\pi G c m_p}{\sigma_e} M_\bullet$$

Writing this in astrophysical scaling

$$L_{Edd} = 1.3\times 10^{39} W\left(\frac{M_\bullet}{10^8 M_\odot}\right) = 3.3\times 10^{12} L_\odot\left(\frac{M_\bullet}{10^8 M_\odot}\right)$$

• So it seems the extreme luminosity of quasars could be explained by super-massive black holes.

• How much mass is required to generate the Eddington luminosity? If the mass falling onto the black hole were *completely converted to energy* then

$$E = mc^2 \qquad \rightarrow \qquad L = \frac{dE}{dt} = c^2\frac{dm}{dt}$$

• Of course, not *all* of the mass is converted to energy; only some fraction η (called the *efficiency* is, so:

$$L = \eta c^2 \frac{dm}{dt} = \eta c^2 \dot{m}$$

• Equating this with the Eddington luminosity gives us the required accretion rate to support the observed luminosity

$$L_{Edd} = \eta c^2 \dot{m} \qquad \rightarrow \qquad \dot{m} = \frac{L_{Edd}}{\eta c^2} = 2\frac{M_\odot}{yr}\left(\frac{M_\bullet}{10^8 M_\odot}\right)\left(\frac{0.1}{\eta}\right)$$

• If the accretion rate grows past this limit, the luminosity increases beyond the Eddington luminosity, and the outward radiative pressure increases shutting off the accretion, regulating the output of the quasar