# Stellar Energy

## Powering the Sun

• We have already looked at several sources of energy, asking if they could power a star, in particular gravitational potential energy and chemical energy. In both instances, these processes cannot produce enough energy to power the stars for a full stellar lifetime.

• These points were realized early on, but some key developments in the late 19th and early 20th Centuries paved the way to the understanding we have today

- ▷ **1896:** Becquerel discovers that uranium has some energetic properties, able to expose film plates near it
- ▷ **1903:** Pierre Curie discovers that radium salts constantly release heat
- ▷ **1904:** Rutherford discovers that radioactive $\alpha$-decay releases energy

• At this point, one could get excited that perhaps radioactive decays are responsible for the energy of the stars, but very little of the heavy, radioactive elements are seen in the spectra of stellar atmospheres. Astrophysicists went back to the drawing boards. . .

- ▷ **1920:** F.W. Aston precisely measures the mass of hydrogen and helium, and discovers that $M_{He} < 4 \cdot m_H$
- ▷ **1920:** Arthur Stanley Eddington immediately grasps the significance — maybe the stars could be powered through *fusion* of nuclei

• Eddington's presumption can be tested without knowing *any* nuclear physics except Aston's result. The key is Einstein's result from special relativity, *energy mass equivalence*, namely: $E = mc^2$

• Generically, for a nucleus with mass $M_{nucleus}$ composed of $Z$ protons and $N$ neutrons (the atomic mass number is $A = Z + N$)

$$M_{nucleus}c^2 = Z \cdot (m_p c^2) + N \cdot (m_n c^2) + BE$$

where the quantity BE is the *nuclear binding energy*: it is the amount of work that has to be done to completely separate the nucleus.

• Before we talk about any nuclear physics, use this result to address Eddington's supposition. This is astrophysics, so we'll have to wave our hands around, and in the absence of all knowledge make some education guesses. The key points in our computation will be:

- ▷ Consider the conversion of hydrogen to helium as the principle energy source, where the energy being released is nuclear binding energy computed using $E = mc^2$ considerations
- ▷ Compute the binding energy of a helium nucleus — this is the amount of energy that is released to form 1 helium nucleus.
- ▷ Assume some fraction of the Sun will ultimately be engaged in this nuclear energy conversion process. It is hottest near the core, so take the inner 10% as a reasonable guess.

## Nuclear Lifetime of the Sun

First we need the energy released by binding four hydrogen atoms to make a helium atom. The masses of interest are:

$$m_H = m_p = 1.67262158 \times 10^{-27}\text{kg}$$

$$m_{He} = 1.67262158 \times 10^{-27}\text{kg}$$

Next we utilize the mass-Energy equivalence and solve for the binding energy

$$BE = (m_{He} - 4 \cdot m_H) \cdot c^2 \qquad \rightarrow \qquad BE = -3.9548 \times 10^{-12}\text{J}$$

This is the amount of energy released for every 4 hydrogen atoms.

The mass of the Sun is $M_\odot = 1.989 \times 10^{30}$ kg, but only 10% will be burning. The number of hydrogen atoms then is

$$N_H = 0.1 \cdot M_\odot / m_p = 1.1892 \times 10^{56}\text{H atoms} \qquad \underbrace{\rightarrow}_{\div 4} \qquad N_{He} = 2.9729 \times 10^{55}\text{He atoms}$$

Multiplying the number of helium atoms by the energy released to make one gives an estimate of the total energy the Sun could expend via this mechanism:

$$E_\odot = N_{He} \cdot |BE| = 1.1757 \times 10^{44}\text{J}$$

If the Sun shines at a constant rate at its current luminosity, then the lifetime would be

$$\Delta t = \frac{E_\odot}{L_\odot} = \frac{1.1757 \times 10^{44}\text{J}}{3.839 \times 10^{26}\text{W}} = 3.06 \times 10^{17}\text{s} = 9.7 \times 10^{9}\text{yr}$$

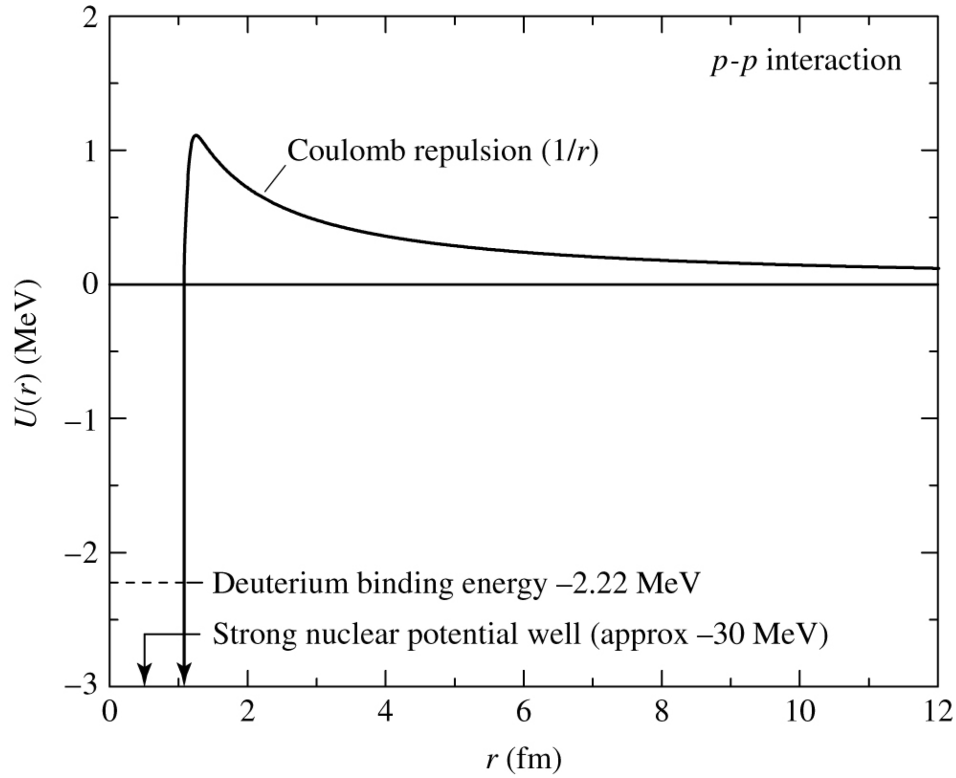
***This is now older than the age of any rocks known, so this is a new upper limit on the age of the Cosmos!***

### Nuclear forces

• Think about a helium nucleus for a moment. It has two neutrons, and two protons. What is holding the protons together? The Coulomb repulsion for two protons separated on nuclear lengthscales ($\ell \sim 1 \times 10^{-15}$ m $\equiv$ 1 fm) is about 230 N. That's *enormous* for sub-atomic particles

• When physicists started thinking about this fact it was clear there must be some new, unknown force counteracting the Coulomb repulsion. This new force is now known as the *strong nuclear force*. It is *short range*, meaning it doesn't have effect significantly outside the atomic nucleus.

• A figure of the potential energy is shown below, showing the steep rise in the Coulomb interaction as two protons approach each other, and the sharp drop created by the sudden turning on of the strong nuclear force.




• How do you get past the Coulomb potential barrier, into the regime where the strong nuclear force dominates? ***Lots of kinetic energy!*** That's why fusion happens in the cores of stars, where the high pressure means high temperatures, and high temperatures mean the average kinetic energy of the particles is enormous.

• We can compute how fast the particles have to be going to overcome the Coulomb repulsion by equating the Coulomb potential energy with the average kinetic energy. Since the kinetic energy is connected to the average temperature of the particles, we can estimate the temperature at the core of the star (see this week's homework!)

## Nuclear reaction chains

• There are several different nuclear reactions that are important in stars. We will state them here by fiat.

• The simplest reaction chain is the *proton-proton chain* that converts hydrogen to helium. Even though this notion was advocated by Eddington in the 1920's, it was not worked out until 1939 by Hans Bethe, a body of work which won him the Nobel Prize in Physics in 1967. Here are the elements in the basic chain:

$$p + p \quad \rightarrow \quad d + e^+ + \nu$$

Here $e^+$ is a positron, $\nu$ is a neutrino, and $d$ is the *deuteron* – a bound proton and neutron. We could have just as easily written this in full nuclear algebra notation as

$$^1_1H +^1_1 H \quad \rightarrow \quad ^2_1H + e^+ + \nu$$

The chain continues as follows:

$$d + p \quad \rightarrow \quad ^3He + \gamma$$

where $\gamma$ is a photon, and $^3He$ is the bound state of two protons and a neutron. Next:

$$^3He +^3 He \quad \rightarrow \quad ^4He + p + p$$

• Note the whole process required 6 protons, but we get two of them back. In the end we are left with a $^4He$, two protons ($p$), two positrons ($e^+$), two neutrinos ($\nu$), and two photons ($\gamma$). The photons we are familiar with; the neutrinos have been a great source of interest in solar physics for the greater part of the Twentieth Century since they are another observational signature that can be used to probe the Sun.

## Heavy star nuclear reaction chains

• In stars much heavier than the Sun, the core temperatures are higher and helium nuclei can fuse in the *triple alpha* process to make *carbon*:

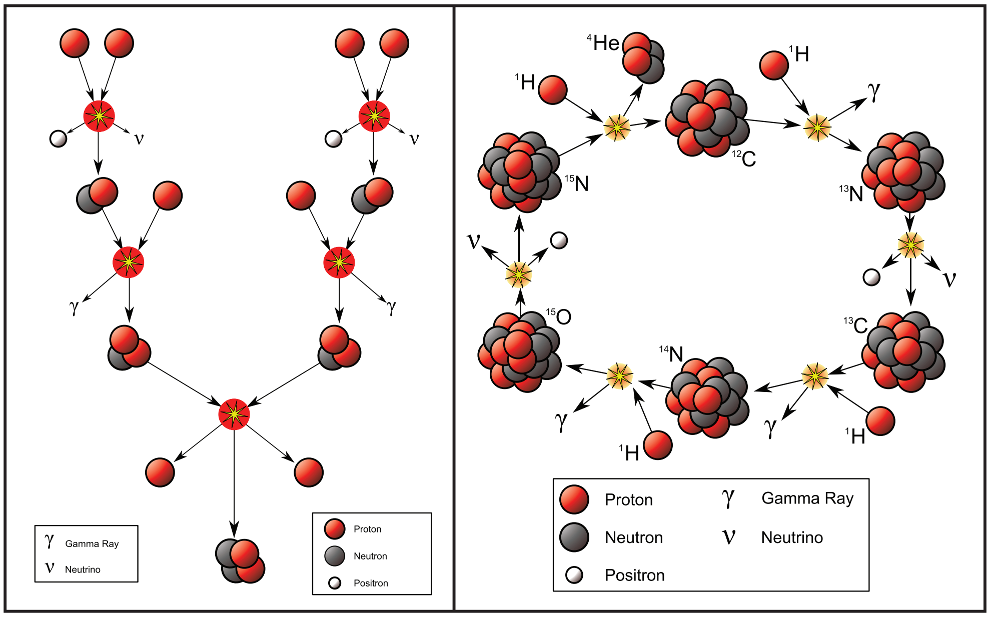
$$^4He +^4 He \quad \rightarrow \quad ^8Be + \gamma$$

• At this stage, if you look at the binding energy of $^8Be$ is *less* than $^4He$. That means $^8Be$ is less stable than $^4He$ and should break up much more readily. Ultimately, statistics plays a role — there are so many alpha particles ($^4He$) flying around, that some $^8Be$ will absorb them before they have a chance to decay, resulting in

$$^8Be +^4 He \quad \rightarrow \quad ^{12}C + \gamma$$

• In Sun-like stars, this carbon generation process becomes important near the end of their lives, but in massive stars the generation of carbon opens another helium production chain known as the CNO cycle:

$$\begin{aligned} ^{12}C + p &\quad \rightarrow \quad ^{13}N + \gamma \\ ^{13}N &\quad \rightarrow \quad ^{13}C + e^+ + \nu \\ ^{13}C + p &\quad \rightarrow \quad ^{14}N + \gamma \\ ^{14}N + p &\quad \rightarrow \quad ^{15}O + \gamma \\ ^{15}O &\quad \rightarrow \quad ^{15}N + e^+ + \nu \\ ^{15}N + p &\quad \rightarrow \quad ^{12}C + 4^{He} \end{aligned}$$




• Can this go on indefinitely? *No.* Nature does what is *energetically favorable.* In terms of nuclear fusion reactions, energy is *released* for all fusion reactions leading up to $^{56}Fe$. The figure shows the binding energy per nucleon, a measure of the stability of the nucleus.

• Note that on the high nuclear mass side of this plot, it is also energetically favorable to climb toward $^{56}Fe$. In this instance, the atoms release energy through *nuclear fission*