Some Basic Properties of Stars

Thinking about what data tells us

• Stars have a variety of properties that can be measured: *brightness, distance, rotation rate, temperature, radius, mass*

• There are a variety of things you can do with any compilation of scientific data

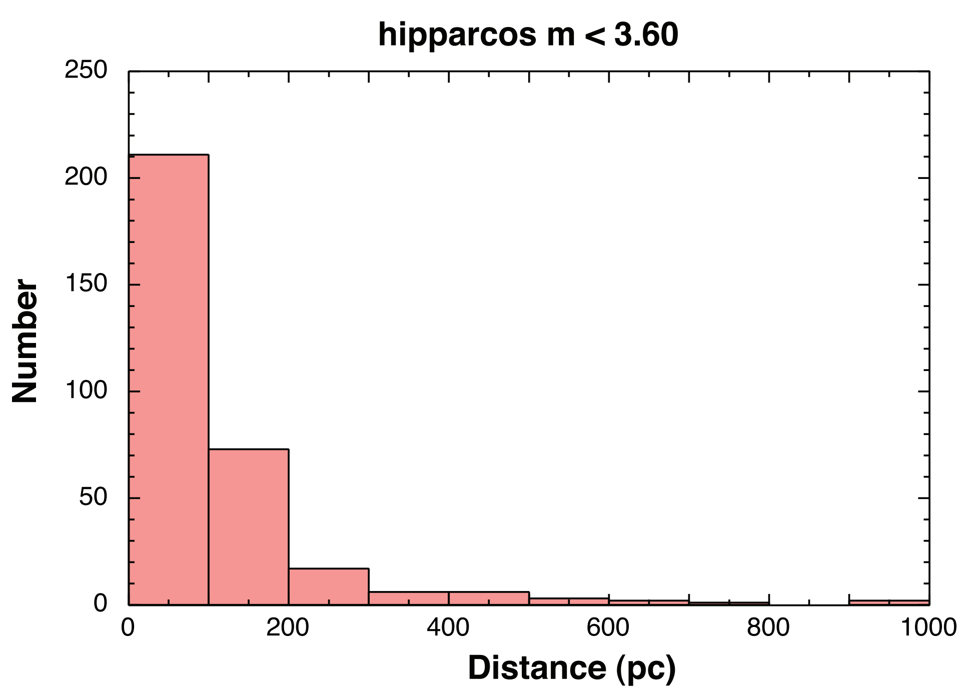
▷ You can take a huge number of values from many different systems and make a histogram of the values. The *shape* of this histogram gives you a sense of the relative likelihood that one value of the parameter is more likely than another, and is related to something called the PDF — the *probability distribution function*

▷ You can look for *correlations* between parameters. There is a formal mathematical definition of correlations, that allow you to make a definitive statistical statement about your data, but at zeroth order the easiest thing to do is make plots of one parameter against another

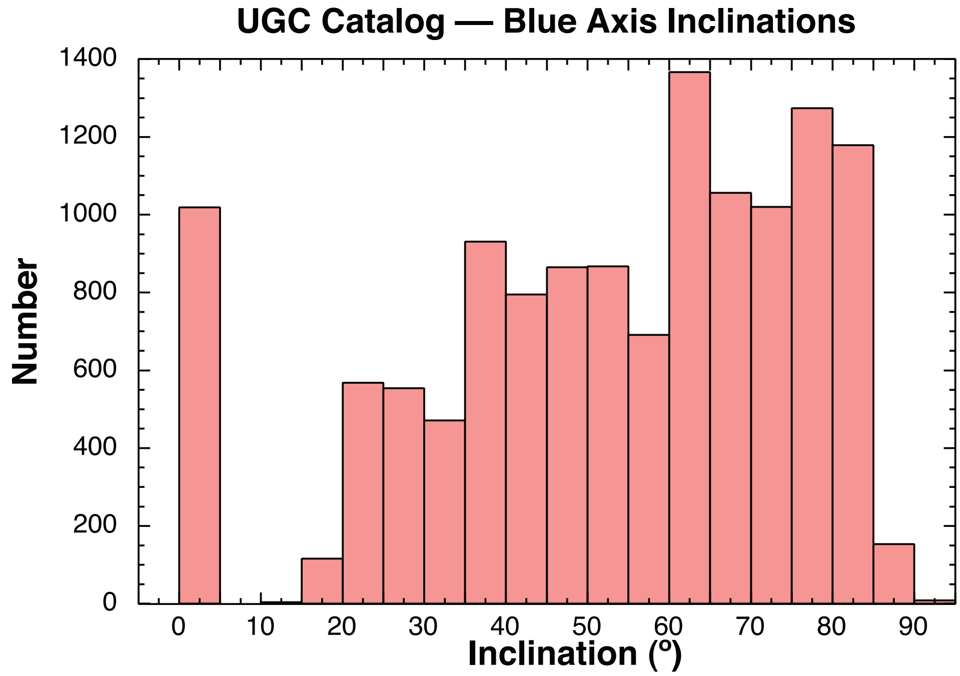
• Sometimes there are good reasons to look for relationships in your data and sometimes not. Sometimes when you are completely ignorant about the nature of your data looking for relationships illuminates previously unknown physics!

• Let's think about some of these examples.

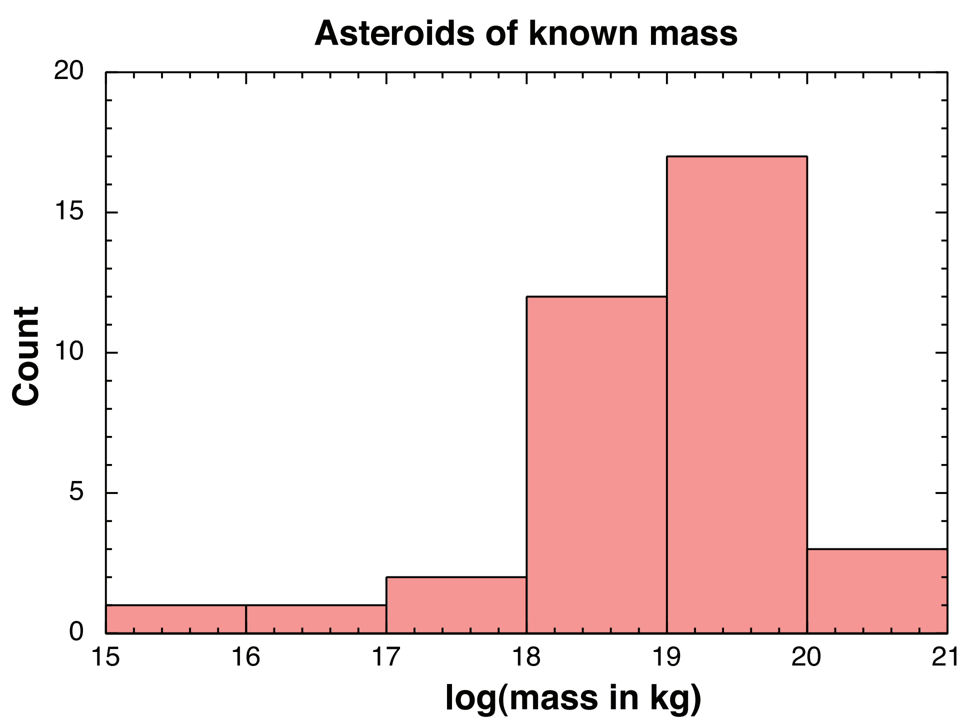
[► **EX 1** ◄]*Distance.* Consider the distance to the 300 brightest stars in the night sky. In what way is distance meaningful?



[► **EX 2** ◄]*Inclination.* Inclination angles of 13,000 galaxies in the UGC galaxy catalog. In what way is inclination meaningful?



[► **EX 3** ◄]*Mass.* Masses of asteroids with known masses. In what way is the value of M meaningful?



• In astrophysics, it is often useful to begin with the question: *does the value of this parameter tell me something about the fundamental nature of the source?* We often separate parameters into two types:

▷ *Extrinsic parameters.* These are parameters whose measured values are dependent on the relation to the observer. If you simply move the observer around in some way, and the value of the parameter changes, then it is extrinsic. Good examples are: distance, inclination angle, apparent magnitude, etc.

▷ *Intrinsic parameters.* These are parameters whose measured values are dependent on the internal physics that governs the astrophysical system. If you change the values of these parameters, and something fundamental happens to the system, then it is intrinsic. Good examples are: mass, temperature, absolute magnitude, spin.

Distribution functions

• What is the PDF? It is really a statement about probability. The PDF $\mathcal{P}(\xi)$ for parameter ξ encodes the relative likelihood for the parameter to take on a certain value. The probability that the parameter has values $a \le \xi \le b$ is

$$P(a \le \xi \le b) = \int_a^b \mathcal{P}(\xi) d\xi \qquad \rightarrow \qquad P(\xi) = \int_{-\infty}^{\infty+} \mathcal{P}(\xi) d\xi = 1$$

• Another way to think about $\mathcal{P}(\xi)$ is in terms of fractions of a population, like asteroids or white dwarfs or plasma ions, then $\mathcal{P}(\xi)$ is *the fraction of all particles that have parameter values in the range of ξ to $\xi + d\xi$.*

[► **EX 1** ◄] *Speed distribution.* Suppose you wanted to simulate the long term evolution of an asteroid belt based on different formation scenarios. In order to set up the initial orbits for the asteroids, you will have to assign them random speeds. You decide to use a *Gaussian distribution*, centered on v_c, the Keplerian speed for a circular orbit at the radius of Earth's asteroid belt. So:

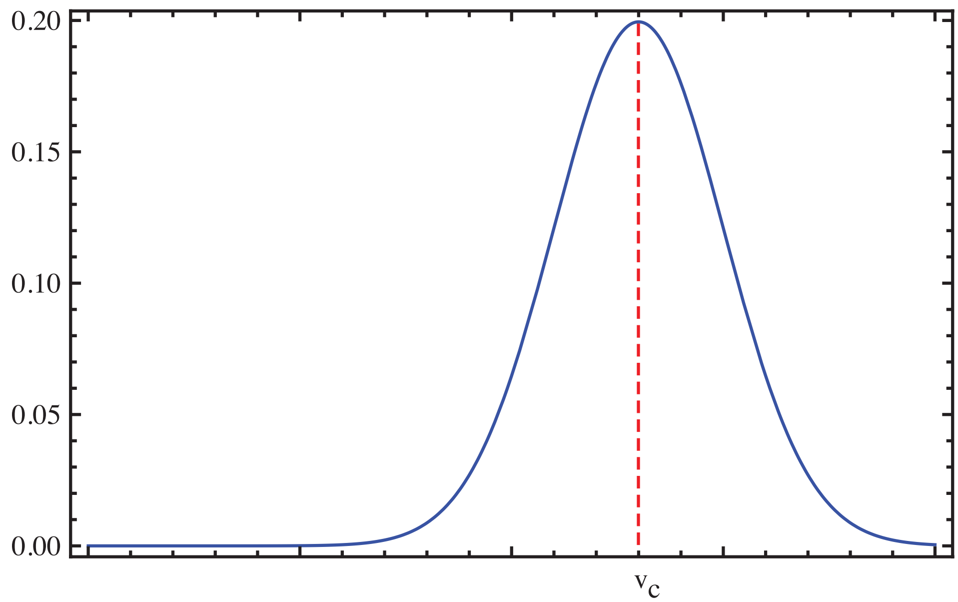
$$\mathcal{P}(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(v - v_c)^2}{2\sigma^2}\right]$$

Note this is just the Gaussian function we learned about before: σ is what you are used to calling the *standard deviation* in statistics; σ^2 is usually called the *variance.* The average of the Gaussian distribution is the value we want to be most likely, in this case $\mu = v_c$

[► **EX 2** ◄] *Initial mass function.* As we will soon learn, mass of a star is the primary driver in stellar evolution. One is very interested to know then how many stars of a given mass exist. The distribution function for the number of stars with masses between m and $m + dm$ is given by

$$\mathcal{P}(m) = \mathcal{M}_o m^\alpha$$

For this to be useful, you have to choose a value of α. The Salpeter IMF (ApJ **121**, 161 [1955]) is $\alpha = -2.35$, and is usually the default assumption in the absence of other data. The constant $\mathcal{M}_o$ is a constant fixed by the number density of the system you are interested in simulating.



EXAMPLE: *The Salpeter IMF can be used to compute the relative numbers of stars of different mass. The Pleiades star cluster (Messier 45) has 19 stars between spectral type B6 and B9. What is the normalization $\mathcal{M}_o$ for the IMF?*

The IMF $\mathcal{P}(m)$ gives the number of stars with mass between m and $m + dm$. So:

$$N = \int_{m_1}^{m_2} \mathcal{P}(m) dm = \int_{m_1}^{m_2} \mathcal{M}_o m^\alpha dm = \mathcal{M}_o \left[\frac{m^{\alpha+1}}{\alpha+1} \right|_{m_1}^{m_2} \qquad \rightarrow \qquad \mathcal{M}_o = \frac{N \cdot (\alpha+1)}{m_2^{\alpha+1} - m_1^{\alpha+1}}$$

The Salpter IMF has $\alpha = -2.35$. Appendix G of the book lists fundamental stellar data for stars of different spectral types. A reasonable interpolation fob B6 adn B9 stars gives $m_1 \simeq 3.0 M_\odot$ and $m_2 \simeq 5.5 M_\odot$. Using these values we find $\mathcal{M}_o = 202.3$.

• How is the PDF useful? In observations it is an important contact point to understand if there is any problem with your observations. If the distribution of your observations do not match the expectation provided by the PDF, there is often an *observational bias* in your data. Biases skew your data away from the true distribution it might have as a consequence of sensitivity or insensitivity to certain characteristics of the systems you are observing (brightness, inclination, color, etc).

• The PDF is also an essential tool in numerical simulation, since I can use it to generate a *realization* that has the appropriate statistical distribution. We will come back to that point later

Measuring the stars

• While the telescope transformed the nature of observational astronomy, the first 200 years of its existence were dominated by its use as a *survey* instrument —- observers plied the skies with their telescopes simply looking for things to add to catalogs of objects in the sky. Today we still carry the legacy of those early efforts with us, most notably in the form of deep sky catalogs such as the famous *Messier Catalog* and J.L.E. Dryer's *New General Catalogue* (the NGC).

• This changed in the 1800's when people began to experiment with attaching *instruments* to telescopes, allowing for precision measurements instead of human approximation. The first instrument was a camera; John William Draper, an amateur astronomer in New York, was the first to do this in 1840, with the Moon as the first target. Eventually, prisms and diffraction gratings were placed between the telescope and the camera, and the first *spectrographs* were born. In 1872, Draper's son Henry was the first person to photographically record the absorption spectrum of a star (Vega).

• The advent of spectroscopy in astronomy put an unprecedented amount of observational power in the hands of scientists. The ability to measure spectra allowed astronomers to acquire data that revealed the composition, motion, and temperature of distant astrophysical systems. Coupled with photographic records of spectra (on glass plates, in those days), this revolution began to produce *prodigious* amounts of data.

• With exploding archives of data, there was a need to have someone look at, organize, and characterize the data (in modern parlance: "they needed some graduate students"). At the Harvard College Observatory, Director Edward Pickering met this need by employing a large group of skilled women to process the astronomical data. This group would become known as the *Harvard Computers*; among them were two women who would transform the way we thought about stellar astronomy and cosmology: Annie Jump Canon and Henrietta Swan Leavitt. We will come back to HSL's story later.

• AJC was put to the task of collecting and organizing stellar spectra. In her life, AJC classified more than $350,000$ spectra in her career! Her scheme for organization was a negotiated compromise between the systems of two other of the Harvard Computers, Williamina Fleming (discoverer of the Horsehead Nebula) and Antonia Maury (who developed the system for identifying giant and dwarf stars) strength of the Balmer absorption lines of hydrogen. She had a series of classes: *A, B, C, . . .* ordered according to her scheme.

• AJC was trying to make sense of stellar properties based on the strength of the hydrogen line. Eventually astronomers figured out how to measure the temperature of the stars using spectroscopy. With this data in hand she noted that the strength of the hydrogen lines did not correspond to the temperature scale. So she shuffled her classification scheme to decreasing temperatures. This gave us the now familiar ordering of spectral classes, *O B A F G K M* ("Oh Be A Fine Guy/Gal Kiss Me", or "Oh Boy Another Fearsome Greedy Klingon Marauder", or "Oh Bother A Fat Gecko Karate Master", or . . .)

Class	*Temperature (K)*	*Mass ($M_\odot$)*	*Radius ($R_\odot$)*	*Luminosity ($L_\odot$)*
O	30000-60000	64	16	1400000
B	10000-30000	18	7	2000
A	7500-10000	3.1	2.1	40
F	6000-7500	1.7	1.4	6
G	5000-6000	1.1	1.1	1.2
K	3500-5000	0.8	0.9	0.4
M	2000-3500	0.4	0.5	0.04

• Being able to measure the temperature of a star was just another parameter in a growing number of stellar properties that could be measured. Astrophysicists had enough data that they were beginning to wonder if they could make sense of it or not. They began to look for correlations in the data.

• Below is the tabulated data for the 20 brightest stars in the sky. Even for just a few stars, it is a *lot* of data to assimilate! How are we going to utilize it? Begin by plotting one value against the other, and then ask yourself if the plot results shows any patterns ("does the plot look interesting?").

Spectral Class and Color

• Characterizing a star by its spectral class (*OBAFGKM*) is fine if you are doing things by hand. But you can't effectively do calculations with the spectral class, nor can you easily convince modern analysis tools (read: spreadsheets) to sort stars into the correct spectral order (*OBAFGKM* as opposed to *ABCDEF...*).

• To this end, astrophysicists have developed a variety of ways to characterize the spectral class. *Temperature* and *color* are foremost among these.

• From a physics viewpoint, the color is most easily characterized by simply measuring the wavelength of the peak emission, λ_{max}. Astronomers, however, favor measuring color in terms of more readily accessible observational quantities like magnitudes.

• The *color index* is defined to be the difference in magnitude of a star seen through two different filters. One of the most popular is the $B - V$ index:

$$B - V = m_B - m_V$$

where B and V are the Johnson-Cousins blue and visual filters respectively

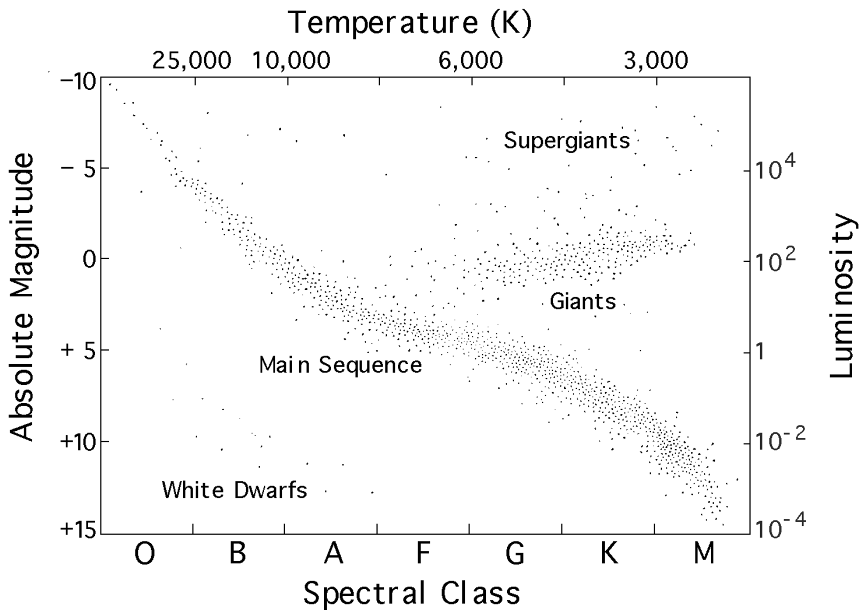
• Meausring the peak wavelength can be mapped onto temperature using Wien's Law:

$$T = \frac{2.90 \times 10^{-3}\mathrm{m} \cdot \mathrm{K}}{\lambda_{max}}$$

Name	***Star***	***Type***	***m***	***M***	***RA (hms)***	***DEC (dms)***	***Parallax ($''$)***	***Distance (pc)***	***Proper Motion ($''$/yr)***	***Radial Motion km/s)***
Sirius	α CMa	A1V	-1.44	1.45	06 45 8.92	-16 41 58	0.37921	2.64	1.33942	-7.7
Canopus	α Car	F0Ib	-0.62	-5.53	06 23 57.11	-52 41 44.4	0.01043	95.88	0.03098	20.5
Arcturus	α Boo	K2II Ip	-0.05	-0.31	14 15 39.67	+19 10 56.7	0.08885	11.26	2.27887	-5.2
Rigel Kentaurus	α Cen	G2V	-0.01	4.34	14 39 36.5	-60 50 2.3	0.74212	1.35	3.70962	-24.6
Vega	α Lyr	A0V	0.03	0.58	18 36 56.34	+38 47 1.3	0.12893	7.76	0.35077	-13.9
Capella	α Aur	M1III	0.08	-0.48	05 16 41.36	+45 59 52.8	0.07729	12.94	0.43375	30.2
Rigel	β Ori	B8Ia	0.18	-6.69	05 14 32.27	-08 12 5.9	0.00422	237	0.00195	20.7
Procyon	α CMi	F5IV-V	0.4	2.68	07 39 18.12	+05 13 30	0.28593	3.5	1.2585	-3.2
Betelgeuse	α Ori	M2Ib	0.45	-5.14	05 55 10.31	+07 24 25.4	0.00763	131	0.02941	21
Achernar	α Eri	B3Vp	0.45	-2.77	01 37 42.85	-57 14 12.3	0.02268	44.09	0.09672	19
Hadar	β Cen	B1III	0.61	-5.42	14 03 49.4	-60 22 22.9	0.00621	161	0.04221	-12
Altair	α Aql	A7IV-V	0.76	2.2	19 50 47	+08 52 6	0.19444	5.14	0.66092	-26.3
Acrux	α Cru	B0.5IV	0.77	-4.19	12 26 35.9	-63 05 56.7	0.01017	98.33	0.03831	-11.2
Aldebaran	α Tau	K5III	0.87	-0.63	04 35 55.24	+16 30 33.5	0.05009	19.96	0.1995	54.1
Spica	α Vir	B1V	0.98	-3.55	13 25 11.58	-11 09 40.8	0.01244	80.39	0.05304	1
Antares	α Sco	M1Ib	1.06	-5.58	16 29 24.46	-26 25 55.2	0.0054	185	0.02534	-3.2
Pollux	β Gem	K0III	1.16	1.09	07 45 18.95	+28 01 34.3	0.09674	10.34	0.62737	3.3
Fomalhaut	α PsA	A3V	1.17	1.74	22 57 39.05	-29 37 20.1	0.13008	7.69	0.3679	6.5
Deneb	α Cyg	A2Ia	1.25	-8.73	20 41 25.91	+45 16 49.2	0.00101	990	0.0022	-4.6
Mimosa	β Cru	B0.5III	1.25	-3.92	12 47 43.26	-59 41 19.5	0.00925	108	0.04991	10.3

HR Diagram

• In 1910 the American astronomer Henry Norris Russell (Nature, **93**, 252 [1914]) and the Danish astronomer Ejnar Hertzsprung (unfortunately, I can't find a citation about this from Hertzsprung!) both discovered a patterned relationship on a digram that now bears their name: the *Hertzsprung Russell (HR) Diagram* plots some measure of brightness on the vertical axis, and some measure of temperature on the horizontal axis.



• One of the most remarkable facts about the HR diagram is that it has this basic appearance for *all populations of stars*. As we will see, one of the most important things about the HR Diagram is it encodes the entire story of evolution of a star, from its birth, through middle age, and to death. It is also valuable for studying stellar systems, particularly clusters.

• For the moment, however, let us consider a more obscure way of using the diagram: to determine distances. You may recall the distance modulus formula:

$$m - M = 5 \cdot \log \left(\frac{d}{10pc} \right)$$

• If we knew both m and M for a star, we can determine its distance. This is where the HR Diagram comes into play — the vertical axis is absolute magnitude, M. But imagine I had a cluster of stars (all born together and located in the same place in space) to which I did not know the distance. I could find the distance in the following way:

- ▷ For the cluster at unknown distance d the distribution of apparent magnitudes m will be the same as for absolute magnitudes M. If I make an HR Diagram using m for the vertical axis, it will *look* like an HR Diagram, just not be calibrated in brightness.
- ▷ Now suppose I have another cluster to which I *do know the distance*. For this cluster, I can make an HR Diagram using absolute magnitude M and it will be correctly calibrated.

▷ If I lay an HR Diagram made with m on top of an HR Diagram made with M, then to make the two main sequences overlap, I have to slide them by an amount $m - M$. *This is just the distance modulus!*

• Consider two clusters in the winter sky, the *Hyades*, and the *Pleiades*. The data for some of the stars in these two clusters is shown below.

PLEIADES

Star	m	B – V
1	9.46	0.47
2	6.29	0.02
3	7.66	0.21
4	6.81	0.06
5	2.87	-0.09
6	10.48	0.64
7	12.22	1.06
8	11.12	0.75
9	13.31	1.45
10	5.45	-0.07
11	8.95	0.39
12	12.26	1.16

HYADES

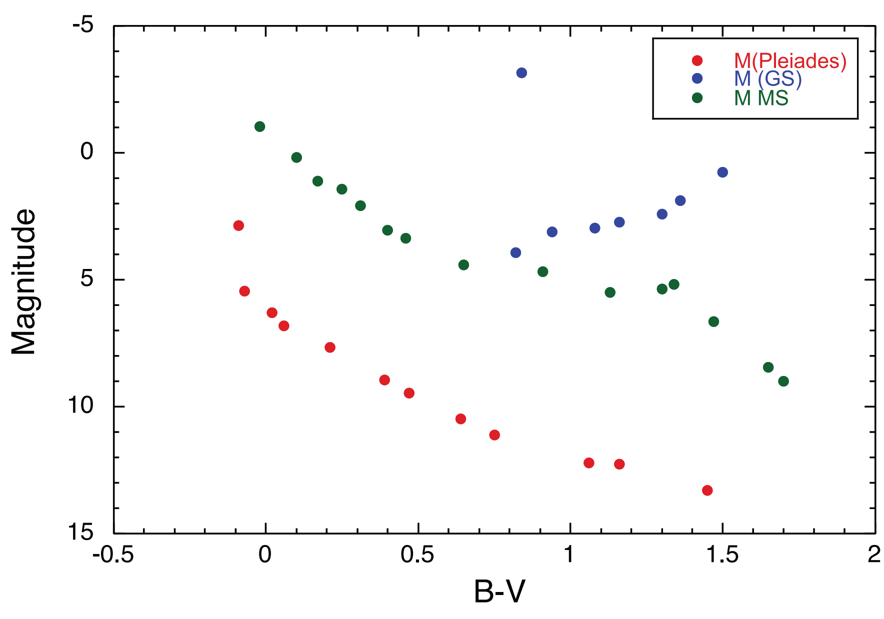
MAIN SEQUENCE STARS

Star	M	B – V
1	3.05	0.40
2	1.12	0.17
3	4.69	0.91
4	5.17	1.34
5	5.37	1.30
6	2.08	0.31
7	8.45	1.65
8	9.00	1.70
9	–1.04	–0.02
10	6.65	1.47
11	0.17	0.10
12	5.50	1.13
13	3.37	0.46
14	1.44	0.25
15	4.41	0.65

GIANT STARS

Star	M	B – V
16	0.77	1.50
17	3.92	0.82
18	–3.15	0.84
19	–2.35	1.55
20	1.89	1.36
21	2.42	1.30
22	2.73	1.16
23	2.96	1.08
24	3.12	0.94

• Making an HR Diagram from this data gives the HR Diagram shown below



• Just eyeballing the offset between the two main sequences, we get $m - M \simeq 6$, so using the distance modulus

$$m - M = 5 \cdot \log\left(\frac{d}{10pc}\right) \qquad \rightarrow \qquad d = 10pc \cdot 10^{(m-M)/5} \sim 160pc$$

• The best distance estimate to the Pleiades is $120 - 140$ pc. Not too shabby for eyeballing it!

Properties across the HR Diagram

• Based on the quantities that we used to make the diagram, it divides itself roughly into quadrants. The top is ***bright*** and the bottom is ***dim***. The left is ***hot*** and the right is ***cool***

• We can use our understanding of other physics and the properties of the diagram itself to also make some important statements about the properties of stars on the diagram. Let's restrict our attention to the Main Sequence

▷ The Main Sequence is an approximate power law with $L \sim T^8$

▷ Stefan-Boltzmann Law tells me that

$$L = 4\pi R_*^2 \sigma T^4$$

▷ Using the luminosity along the main sequence, I can solve for the radius of my star

$$R_* \sim \frac{\sqrt{L}}{T^2} \qquad \rightarrow \qquad L \sim T^8 \qquad \rightarrow \qquad R_* \sim T^2$$

• So the *size* of the star grows with temperature! The hottest main sequence stars are roughly $T \sim 5T_\odot$, and the coolest main sequence stars are roughly $T \sim (1/2)T_\odot$. Thus our expectation for the radii of stars on the main sequence is

$$\frac{1}{4}R_\odot \lesssim R_* \lesssim 25R_\odot$$