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# Some Basic Principles from Astronomy

## The Big Question

One of the most difficult things in every physics class you will ever take is putting what you are learning in context — what is this good for? how do I use it?

I find it useful to have a *Big Question* in mind as we embark on our journey — a central unifying concept that we constantly come back to in an attempt to try and understand how the subject at hand helps us address a specific problem of interest. There are many such questions we could use in astrophysics, but I think a particularly nice one is this:

*What is the size and/or age of the Cosmos?*

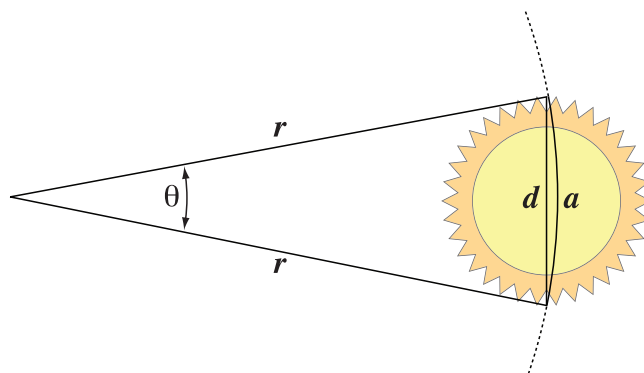
In class, there are specific instances where I will return this question and ask if we can add a new number to our matrix of numbers attempting to answer this important question. If you think of something clever based on what we are learning, please don't hesitate to pipe up or send me an email — we'll add it to our list.

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## Measuring the sky

- The most difficult problem in astronomy is we are confined (for the most part) to the surface of the Earth, and what we would like to know more about is *out there* – thataway! **► Imagine I am gesturing vaguely at the ceiling as you read this ◄**
- In many ways, the story of astronomy began with attempting to measure distances — how big is the Earth? how far away is the Moon? how big is the astronomical unit?
- The ancient Greeks were very good at figuring out how to estimate these values by making simple measurements here on the Earth, then bootstrapping out into the Cosmos using geometry and clever observations. This was the beginning of the development of the *distance ladder*, a concept we will return to when we talk about Cosmology
- The first step to understanding how to measure the Cosmos is to ask “What can I easily measure?” The answer to this question is: *time*, and *angles* (though the former was not possible until the invention of timekeeping devices, like water clocks and cell phones).
- You can construct clever devices to measure angles if you know some trigonometry and can do complicated math like division. One way is using a device called a *cross staff*, and another way is to use a device called *your hands*.
- The way these devices work is you know something about how large they appear when held up and compared against the sky. The fundamental size we begin with is *angular size* — how big of an angle do I have to move my eye to look from one edge of the object to the other on the sky?

- The image below shows the concept of the angular size  $\theta$  for the Sun.



- If I knew the distance  $r$  to the Sun, I could calculate its *physical size*  $d$  using the small angle formula. Remember that for small angles  $\alpha$  I can approximate  $\sin \alpha \sim \alpha$ , so:

$$\sin\left(\frac{\theta}{2}\right) = \frac{d/2}{r} = \frac{d}{2r} \quad \rightarrow \quad \frac{\theta}{2} \simeq \frac{d}{2r} \quad \rightarrow \quad \theta \simeq \frac{d}{r}$$

- Note that if the Sun were far away, so the angle  $\theta$  were very small, then the small angle formula is just the arclength formula,  $\theta = a/r$ . For objects that are close, however, the distance  $a$  may not be well approximated by the distance  $d$ . This is a *choice* that you have to make as an astrophysicist opting to use an approximation: how much error are you willing to accept based on the approximations you are using?

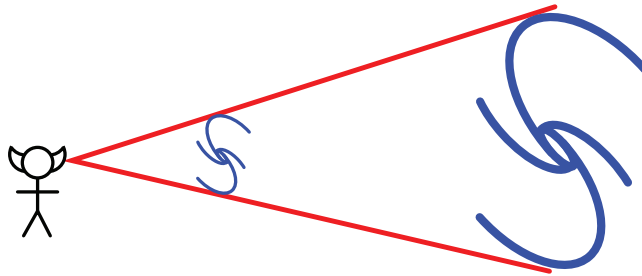
- **NB: These angles are all measured in *radians*!**

- The table below shows the angular size of some astrophysical systems you could observe and measure on your own. Note that I have not chosen a *consistent* system of units for all of these; I have instead reported them in the units conventional for each source.

<i>Object</i>	<i>Angular Size</i>
Moon	$\sim 0.5^\circ$
Sun	$\sim 0.5^\circ$
Mars (Opposition)	$\sim 25''$
Jupiter (Opposition)	$\sim 50''$
Albireo (separation between components)	$35''$
M45 (Pleiades open cluster)	$110'$
M13 (globular cluster)	$20'$
M31 (Andromeda Galaxy)	$60' \times 190'$
Max spot size on CMB	$1^\circ$

- Numbers like these are *easy* to measure. The trick is getting to the *distance*, because that fixes the physical size of the object, as the image below illustrates. At the start of the Twentieth Century, we knew the angular size of the Andromeda Nebula (as M31 was known

then), but the distance to M31 had enormous implications for its physical size — this was the central argument in the famous *Shapley-Curtis Debate*, which we will return to when we study galaxies.



- Just because we don't know the exact value of a distance in astronomy doesn't mean we can't use it in calculations! For instance: we call the distance from the Earth to the Sun an *astronomical unit*: 1 AU. Kepler's Third Law of Motion tells me that the square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit:  $P^2 \propto a^3$

▷ I know the orbit of Jupiter around the Sun has a period  $P_J = 11.8618$  years. If the Earth has an orbital period of  $P_{\oplus} = 1$  year and a semi-major axis of  $a_{\oplus} = 1$  AU, then

$$P^2 \propto a^3 \quad \rightarrow \quad \left(\frac{P_J}{P_{\oplus}}\right)^2 = \left(\frac{a_J}{a_{\oplus}}\right)^3 \quad \rightarrow \quad a_J = a_{\oplus} \left(\frac{P_J}{P_{\oplus}}\right)^{2/3}$$

- So Jupiter must have an orbital size of  $a_J = 1\text{AU} \cdot (11.8618)^{2/3} = 5.20$  AU. Did I know the length of the astronomical unit to do this? *No!* But now I know the distance to Jupiter in terms of the AU, so once I know the length of the AU I'll know the physical distance to Jupiter.

- The AU has an accepted value of 149,597,870.7 km. It plays a role in the definition of another important astronomical distance unit: the *parsec*. The parsec (pc) is defined in terms of a right triangle where

$$\tan(1'') = \frac{1\text{AU}}{1\text{parsec}}$$

- Converting the  $1''$  to radians, one finds that  $1\text{pc} = 206,265$  AU. In modern units,  $1\text{pc} = 3.26156\text{lyr}$ . More often this is derived in terms of *parallax angles* (which you can read all about in CH 3 of BOB), but it is equivalent to this definition if one employs the small angle formula, which is valid for astronomically long skinny triangles. The generic definition of the parsec is *the distance at which 1 AU subtends 1''*.

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## Luminosity and Flux

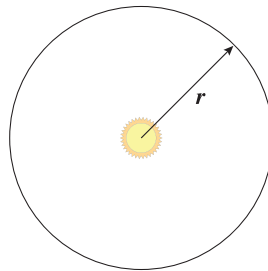
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- When you first took physics in second grade, you learned about the concept of *power* — the amount of energy per unit time that a system generates.

- In astrophysics we spend a lot of time looking at *luminous* objects, and so we like to characterize them in terms of their radiant power, which astronomers call the *luminosity*,  $L$ . The luminosity is the *total energy per unit time* given off by an object.

- Sitting here on Earth looking at a luminous object like a star, are we receiving the total energy it emits? *No!* We are only seeing a small fraction of the energy it emits, defined by the amount of energy that falls on your detector (your eye, or your telescope). What you measure is call the *flux*,  $F$  — the energy per unit time per area:  $F = L/A$

- What is the area  $A$  of interest here? The area that the luminosity is spread across! If I wanted to capture all the energy from the star, I would build a sphere at some radius  $r$  as shown in the figure below (this is called a *Dyson Sphere*)



- The area of the sphere is  $A = 4\pi r^2$ , so the flux  $F$  measured on the sphere is

$$F = \frac{L}{4\pi r^2}$$

- This is the famous  $1/r^2$  law for light! The intensity of the light decreases in proportion to the square of the distance from the source of light.

### The Big Question

Suppose I have an object for which the luminosity,  $L$ , is known. I sit here on Earth, and I measure the flux,  $F$ , which allows me to compute the distance  $r$  to the object!

[► EX ◀] We'd like to know the size of my back field, and we don't have a tape measure. Fortunately, we have a flashlight with four 1 Watt LEDs and a light meter. One night we stand on opposite sides of my field, and I shine the light at you. You measure  $9.46 \times 10^{-5} \text{ W/m}^2$  of flux. How big is my back field?

$$F = \frac{L}{4\pi r^2} \quad \rightarrow \quad r = \sqrt{\frac{L}{4\pi F}} \quad \rightarrow \quad r = 58\text{m}$$

What's the take-away here? If I knew the luminosity of an enormously distant object, like a star or a quasar or some other astrophysical object, measuring the flux for that object defines a lower limit on the radius,  $r$ , of the observable Universe. We will return to this.

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## Magnitudes

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- Astronomers are creatures of habit and tradition. This is because for so long we didn't really know anything, and bootlegged our knowledge on numbers that we would "figure out later" (like the Jupiter calculation we noted above). Magnitudes as a measure of brightness are a part of our long, historical traditions, but they have been formalized so that quantitative computations may be done with them
- The magnitude scale was created by the Greek astronomer Hipparchus who histogrammed the brightness of all the stars he could see in the sky (about 850 total). The brightest stars were dumped in a bin called 1 (magnitude  $m = 1$ ), and the dimmest stars were dumped in a bin called 6 (magnitude  $m = 6$ ). The bins in the middle were filled by stars that differed in brightness by roughly a factor of 2 (this was all done by naked eye).
- The modern mathematical definition is that a difference of 5 magnitudes ( $\Delta m = 6 - 1 = 5$ ) is exactly a change in brightness of a factor of 100. Between each magnitude, the brightness change then is only  $100^{1/5} \simeq 2.512$ . What do I mean when I say the brightness changes by a factor? I mean that the *ratio* of the brightness (where for our purposes flux  $F$  is a measure of the brightness) changes by a factor of 100 for every 5-magnitude change, or the *flux ratio* is

$$\frac{F_1}{F_2} = 100^{(m_2 - m_1)/5}$$

- There are a few things you should keep in mind here:
  - ▷ This is how bright things look when you observe them from Earth, what we call the *apparent magnitude*  $m$ . The brightness of any object will change if I change its distance, so this does not represent its *intrinsic* brightness.
  - ▷ We haven't made any comment on how bright a source is at a given wavelength  $\lambda$  of light! As such, this is called the *bolometric* magnitude, the brightness integrated across all wavelengths. Later we will talk about *band magnitudes* — the brightness over certain limited regions of the spectrum.
  - ▷ This scale is normalized by the stars that Hipparchus observed, namely the naked eye stars. The concept of brightness could apply to anything, however, and astronomers use magnitudes as tools to describe the brightness of *everything*, whether it is a star or not, notably the Moon, Sun and planets, which are enormously brighter than the stars. Very bright objects have small magnitude numbers, *including negative values!* The table below lists some common magnitude values.

<i>Object</i>	<i>Apparent Magnitude</i>	<i>Object</i>	<i>Apparent Magnitude</i>
Sun	-26.73	Moon	-12.74
Sirius	-1.47	Jupiter	$\sim -2.0$
Naked Eye Limit	$\sim 6.5$	M31	3.44
Binocular Limit	$\sim 9.5$	Typical Galaxy	$\sim 11.5$
Brightest Quasar (3C273)	12.91	HST Limit	$\sim 30$

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- Do we care about the apparent magnitude  $m$  of a star in our sky? From the viewpoint of an observer, certainly because the apparent magnitude is what is directly important to your ability to see the star from the Earth. But from the viewpoint of an astrophysicist, trying to understand the fundamental nature of the sources the apparent magnitude is not as useful.

- Consider an example: the brightest star in the sky is Sirius ( $\alpha$  CMa), with an apparent magnitude  $m = -1.47$ . The second brightest is the star Canopus ( $\alpha$  Car) in the southern constellation Carina, has an apparent magnitude of  $m = -0.72$ .

- What should I conclude about the nature of these two stars? Is Sirius brighter than Canopus? That question is impossible to answer without knowing how far it is to the two stars. If Sirius and Canopus were precisely the same distance from the Earth, Canopus would be  $600\times$  brighter!

- To deal with this difficulty, astronomers define the *absolute magnitude*  $M$  to be the brightness any star would have if it were at a fixed distance of  $d = 10$  pc from the Earth. If we denote  $F_{10}$  to be the flux from a star if it were at 10 pc, then we can use the definition of the flux ratio to relate the apparent magnitude  $m$  (for a star at distance  $d$ ) to the absolute magnitude  $M$ :

$$100^{(m-M)/5} = \frac{F_{10}}{F} \quad \rightarrow \quad F = \frac{L}{4\pi d^2} \quad \rightarrow \quad 100^{(m-M)/5} = \left( \frac{d}{10pc} \right)^2$$

- With a little algebra, this becomes the famous *distance modulus* formula:

$$m - M = 5 \cdot \log \left( \frac{d}{10pc} \right)$$

- **NB:** The argument of a logarithm, just like the argument of a trig function, must be *dimensionless!* That means when you are using a formula like the distance modulus formula, where a scaling has been provided in specific units (e.g.  $d/(10 \text{ pc})$ ), you had better use the same units!

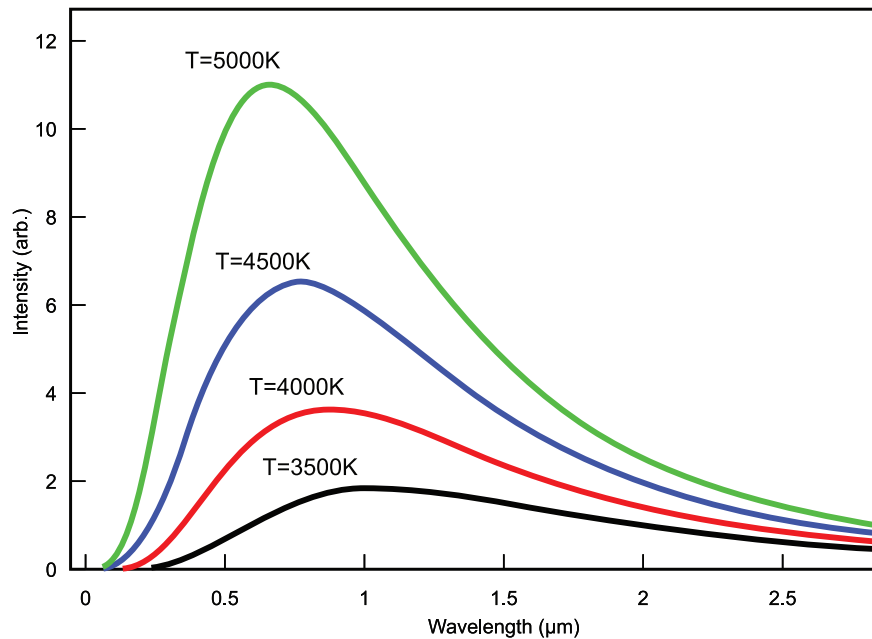
## Blackbodies, Temperature & Color

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- Your eye is sensitive to many different wavelengths of light, as are most common detectors (“detector” is a fancy astronomer word for “camera” — always say “detector” when hanging around biologists, to sound more intimidating). When you look at something, the brightness you perceive is really the integration of the brightness over the many wavelengths to which you are sensitive.

- You may recall from your first encounter with thermodynamics the concept of a *blackbody*. A blackbody is a body in thermodynamic equilibrium with its environment — it absorbs and emits radiation so there is no net flow of energy. The operational definition is that a blackbody absorbs all radiation that strikes it. Stars are good approximate blackbodies — go shine your laser pointer at the Sun and tell me how much is reflected back at you.

- The intensity spectrum for several blackbodies are shown in the figure below.



- Once we had the technological ability to measure the intensity as a function of wavelength, it became clear that the peak emission wavelength,  $\lambda_{max}$ , is a function of temperature  $T$ , as the figure shows. The value of  $\lambda_{max}$  is given by *Wien's Displacement Law*, an empirical result derived by fitting the peaks

$$\lambda_{max} = \frac{2.90 \times 10^{-3} \text{m} \cdot \text{K}}{T}$$

- How is this useful? It allows you (for instance) to determine the surface temperature of the stars by simply measuring the peak emission wavelength! This is one of the fundamental principles of stellar astrophysics — temperature and color are *correlated*. We can (and will) sometimes use them interchangeably.

- The peak wavelength of the Sun is  $\lambda_{max} = 500 \text{ nm}$  (fire-engine green). Using Wien's Displacement Law gives an effective temperature of  $T = 5800 \text{ K}$ .

- The intensity spectra shown above represent the energy emitted at every wavelength. If you integrate over the curve, you get the total energy emitted by the blackbody. You may recall that the total radiant power (luminosity,  $L$ ) is given by the Stefan-Boltzmann Law, which is also dependent on the temperature. For a star of radius  $R$ :

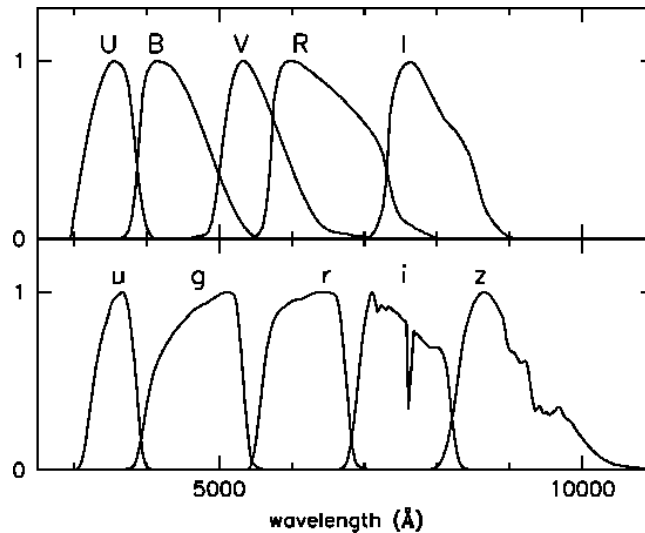
$$L = A_{emit} \sigma T^4 \quad \rightarrow \quad L = 4\pi R^2 \sigma T^4$$

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## Color Index

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- Astronomers use standard photometric filters to restrict attention to only particular parts of the electromagnetic spectrum. There are two basic standards: the *Johnson-Cousins* (UBVRI) and the *Sloan* (ugriz). The wavelengths that each of these filters pass are shown in the image below.



- For a variety of reasons, it is often useful to characterize a star not with a single color, but with two. The *color index* is defined to be the difference in the absolute magnitude of a star measured in two different filters. For instance, the  $B - V$  color index is

$$B - V = M_B - M_V$$

- Note that the color index can be positive or negative, and simply characterizes the relative contributions of different colors to a star's brightness